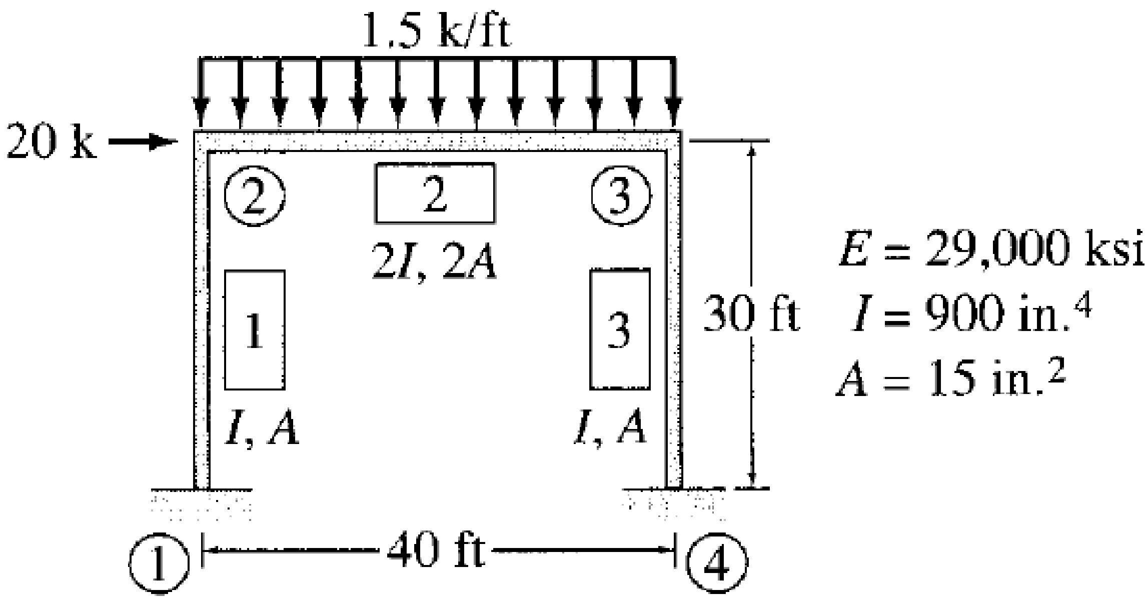
Ejemplo del Método General de las Rigideces, aplicado en Marcos



Encuentre las reacciones y las fuerzas en los extremos de los miembros en las coordenadas locales para el marco mostrado, empleando el Método Matricial de las Rigideces.

0. Información de entrada

$$A := \begin{bmatrix} \frac{15}{12^2} \\ \frac{30}{12^2} \\ \frac{15}{12^2} \end{bmatrix} ft^2 \qquad I := \begin{bmatrix} \frac{900}{12^4} \\ \frac{1800}{12^4} \\ \frac{900}{12^4} \end{bmatrix} ft^4 \qquad E := \begin{bmatrix} 29000 \cdot 12^2 \\ 29000 \cdot 12^2 \\ 29000 \cdot 12^2 \end{bmatrix} ksf \qquad L := \begin{bmatrix} 30 \\ 40 \\ 30 \end{bmatrix} ft$$

$$x := \begin{bmatrix} 0 & 0 \\ 0 & 40 \\ 40 & 40 \end{bmatrix} \qquad y := \begin{bmatrix} 0 & 30 \\ 30 & 30 \\ 0 & 30 \end{bmatrix}$$

$$N := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad V := \begin{bmatrix} 0 & 0 \\ 30 & 30 \\ 0 & 0 \end{bmatrix} \qquad M := \begin{bmatrix} 0 & 0 \\ 200 & -200 \\ 0 & 0 \end{bmatrix}$$

$$\frac{1.5 \cdot 40}{2} = 30$$

$$\frac{1.5 \cdot 40^2}{12} = 200$$

1. Matrices de rigidez de cada barra

1.1 Barra 1

Fórmulas

Matriz de rigidez de barra, considerando efectos axiales, en el SCL.

$$k(n) := \frac{E_n \cdot I_n}{{L_n}^3} \cdot \begin{bmatrix} \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & 12 & 6 \cdot L_n & 0 & -12 & 6 \cdot L_n \\ 0 & 6 \cdot L_n & 4 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 2 \cdot (L_n)^2 \\ -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & -12 & -6 \cdot L_n & 0 & 12 & -6 \cdot L_n \\ 0 & 6 \cdot L_n & 2 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 4 \cdot (L_n)^2 \end{bmatrix}$$

Matriz de transformación

$$\mathrm{seno}(n) := \frac{y_{n\,2} - y_{n\,1}}{L_n} \qquad \mathrm{coseno}(n) := \frac{x_{n\,2} - x_{n\,1}}{L_n}$$

$$T(n) := \begin{bmatrix} \mathrm{coseno}(n) & \mathrm{seno}(n) & 0 & 0 & 0 & 0 \\ -\mathrm{seno}(n) & \mathrm{coseno}(n) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{coseno}(n) & \mathrm{seno}(n) & 0 \\ 0 & 0 & 0 & -\mathrm{seno}(n) & \mathrm{coseno}(n) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriz de rigidez de barra, considerando efectos axiales, en el SCG.

$$K(n) := T(n)^T \cdot k(n) \cdot T(n)$$ NO ES MULTIPLICACIÓN

Vector de fuerzas debidas a las cargas dentro de los miembros, SCL.

$$Q_f(n) := \begin{bmatrix} N_{n\,1} \\ V_{n\,1} \\ M_{n\,1} \\ N_{n\,2} \\ V_{n\,2} \\ M_{n\,2} \end{bmatrix}$$

Vector de fuerzas debidas a las cargas dentro de los miembros, SCG.

$$F_f(n) := T(n)^{-1} \cdot Q_f(n)$$

$$k_1 := k(1) = \begin{bmatrix} 14500 & 0 & 0 & -14500 & 0 & 0 \\ 0 & 80.5556 & 1208.3333 & 0 & -80.5556 & 1208.3333 \\ 0 & 1208.3333 & 24166.6667 & 0 & -1208.3333 & 12083.3333 \\ -14500 & 0 & 0 & 14500 & 0 & 0 \\ 0 & -80.5556 & -1208.3333 & 0 & 80.5556 & -1208.3333 \\ 0 & 1208.3333 & 12083.3333 & 0 & -1208.3333 & 24166.6667 \end{bmatrix}$$

$$T_1 := T(1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$K_1 := K(1) = \begin{bmatrix} 80.5556 & 0 & -1208.3333 & -80.5556 & 0 & -1208.3333 \\ 0 & 14500 & 0 & 0 & -14500 & 0 \\ -1208.3333 & 0 & 24166.6667 & 1208.3333 & 0 & 12083.3333 \\ -80.5556 & 0 & 1208.3333 & 80.5556 & 0 & 1208.3333 \\ 0 & -14500 & 0 & 0 & 14500 & 0 \\ -1208.3333 & 0 & 12083.3333 & 1208.3333 & 0 & 24166.6667 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

1.2 Barra 2

$$k_2 := k(2) = \begin{bmatrix} 21750 & 0 & 0 & -21750 & 0 & 0 \\ 0 & 67.9688 & 1359.375 & 0 & -67.9688 & 1359.375 \\ 0 & 1359.375 & 36250 & 0 & -1359.375 & 18125 \\ -21750 & 0 & 0 & 21750 & 0 & 0 \\ 0 & -67.9688 & -1359.375 & 0 & 67.9688 & -1359.375 \\ 0 & 1359.375 & 18125 & 0 & -1359.375 & 36250 \end{bmatrix}$$

$$T_2 := T(2) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$K_2 := K(2) = \begin{bmatrix} 21750 & 0 & 0 & -21750 & 0 & 0 \\ 0 & 67.9688 & 1359.375 & 0 & -67.9688 & 1359.375 \\ 0 & 1359.375 & 36250 & 0 & -1359.375 & 18125 \\ -21750 & 0 & 0 & 21750 & 0 & 0 \\ 0 & -67.9688 & -1359.375 & 0 & 67.9688 & -1359.375 \\ 0 & 1359.375 & 18125 & 0 & -1359.375 & 36250 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

1.3 Barra 3

$$k_3 := k(3) = \begin{bmatrix} 14500 & 0 & 0 & -14500 & 0 & 0 \\ 0 & 80.5556 & 1208.3333 & 0 & -80.5556 & 1208.3333 \\ 0 & 1208.3333 & 24166.6667 & 0 & -1208.3333 & 12083.3333 \\ -14500 & 0 & 0 & 14500 & 0 & 0 \\ 0 & -80.5556 & -1208.3333 & 0 & 80.5556 & -1208.3333 \\ 0 & 1208.3333 & 12083.3333 & 0 & -1208.3333 & 24166.6667 \end{bmatrix}$$

$$T_3 := T(3) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 4 & 5 & 6 \end{bmatrix}$$

$$K_3 := K(3) = \begin{bmatrix} 80.5556 & 0 & -1208.3333 & -80.5556 & 0 & -1208.3333 \\ 0 & 14500 & 0 & 0 & -14500 & 0 \\ -1208.3333 & 0 & 24166.6667 & 1208.3333 & 0 & 12083.3333 \\ -80.5556 & 0 & 1208.3333 & 80.5556 & 0 & 1208.3333 \\ 0 & -14500 & 0 & 0 & 14500 & 0 \\ -1208.3333 & 0 & 12083.3333 & 1208.3333 & 0 & 24166.6667 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

2. Vectores de carga

2.1 Barra 1

$$Q_{f1} := Q_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad F_{f1} := F_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

2.2 Barra 2

$$Q_{f2} := Q_f(2) = \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix} \qquad F_{f2} := F_f(2) = \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

2.3 Barra 3

$$Q_{f3} := Q_f(3) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad F_{f3} := F_f(3) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

3. Ensamblaje de la matriz de rigidez global del sistema

(Ensamblaje manual)

$$S := \begin{bmatrix} K1_{4\,4}+K2_{1\,1} & K1_{4\,5}+K2_{1\,2} & K1_{4\,6}+K2_{1\,3} & K2_{1\,4} & K2_{1\,5} & K2_{1\,6} \\ K1_{5\,4}+K2_{2\,1} & K1_{5\,5}+K2_{2\,2} & K1_{5\,6}+K2_{2\,3} & K2_{2\,4} & K2_{2\,5} & K2_{2\,6} \\ K1_{6\,4}+K2_{3\,1} & K1_{6\,5}+K2_{3\,2} & K1_{6\,6}+K2_{3\,3} & K2_{3\,4} & K2_{3\,5} & K2_{3\,6} \\ K2_{4\,1} & K2_{4\,2} & K2_{4\,3} & K2_{4\,4}+K3_{4\,4} & K2_{4\,5}+K3_{4\,5} & K2_{4\,6}+K3_{4\,6} \\ K2_{5\,1} & K2_{5\,2} & K2_{5\,3} & K2_{5\,4}+K3_{5\,4} & K2_{5\,5}+K3_{5\,5} & K2_{5\,6}+K3_{5\,6} \\ K2_{6\,1} & K2_{6\,2} & K2_{6\,3} & K2_{6\,4}+K3_{6\,4} & K2_{6\,5}+K3_{6\,5} & K2_{6\,6}+K3_{6\,6} \end{bmatrix}$$

$$S = \begin{bmatrix} 21830.5556 & 0 & 1208.3333 & -21750 & 0 & 0 \\ 0 & 14567.9688 & 1359.375 & 0 & -67.9688 & 1359.375 \\ 1208.3333 & 1359.375 & 60416.6667 & 0 & -1359.375 & 18125 \\ -21750 & 0 & 0 & 21830.5556 & 0 & 1208.3333 \\ 0 & -67.9688 & -1359.375 & 0 & 14567.9688 & -1359.375 \\ 0 & 1359.375 & 18125 & 1208.3333 & -1359.375 & 60416.6667 \end{bmatrix}$$

4. Ensamblaje de los vectores de fuerzas

(Ensamblaje manual)

$$P := \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$P_f := \begin{bmatrix} F_{f1_4}+F_{f2_1} \\ F_{f1_5}+F_{f2_2} \\ F_{f1_6}+F_{f2_3} \\ F_{f3_4}+F_{f2_4} \\ F_{f3_5}+F_{f2_5} \\ F_{f3_6}+F_{f2_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix}$$

$$P - P_f = \begin{bmatrix} 20 \\ -30 \\ -200 \\ 0 \\ -30 \\ 200 \end{bmatrix}$$

5. Solución del sistema de ecuaciones

$$d := S^{-1} \cdot \left(P - P_f\right) = \begin{bmatrix} 0.1621 \\ -0.0016 \\ -0.0072 \\ 0.1613 \\ -0.0025 \\ 0.0022 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

SCG

6. Fuerzas en los miembros

6.1 Barra 1

$$v_1 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1621 \\ -0.0016 \\ -0.0072 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$u_1 := T(1) \cdot v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0016 \\ -0.1621 \\ -0.0072 \end{bmatrix}$$

$$Q_1 := k_1 \cdot u_1 + Q_{f1} = \begin{bmatrix} 23.2563 \\ 4.3023 \\ 108.295 \\ -23.2563 \\ -4.3023 \\ 20.7747 \end{bmatrix}$$

$$F_1 := K_1 \cdot v_1 + F_{f1} = \begin{bmatrix} -4.3023 \\ 23.2563 \\ 108.295 \\ 4.3023 \\ -23.2563 \\ 20.7747 \end{bmatrix}$$

6.2 Barra 2

$$v_2 := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0.1621 \\ -0.0016 \\ -0.0072 \\ 0.1613 \\ -0.0025 \\ 0.0022 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$u_2 := T(2) \cdot v_2 = \begin{bmatrix} 0.1621 \\ -0.0016 \\ -0.0072 \\ 0.1613 \\ -0.0025 \\ 0.0022 \end{bmatrix}$$

$$Q_2 := k_2 \cdot u_2 + Q_{f2} = \begin{bmatrix} 15.6977 \\ 23.2563 \\ -20.7747 \\ -15.6977 \\ 36.7437 \\ -248.9724 \end{bmatrix}$$

$$F_2 := K_2 \cdot v_2 + F_{f2} = \begin{bmatrix} 15.6977 \\ 23.2563 \\ -20.7747 \\ -15.6977 \\ 36.7437 \\ -248.9724 \end{bmatrix}$$

6.3 Barra 3

$$v_3 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1613 \\ -0.0025 \\ 0.0022 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$u_3 := T(3) \cdot v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0025 \\ -0.1613 \\ 0.0022 \end{bmatrix}$$

$$Q_3 := k_3 \cdot u_3 + Q_{f3} = \begin{bmatrix} 36.7437 \\ 15.6977 \\ 221.9578 \\ -36.7437 \\ -15.6977 \\ 248.9724 \end{bmatrix}$$

$$F_3 := K_3 \cdot v_3 + F_{f3} = \begin{bmatrix} -15.6977 \\ 36.7437 \\ 221.9578 \\ 15.6977 \\ -36.7437 \\ 248.9724 \end{bmatrix}$$