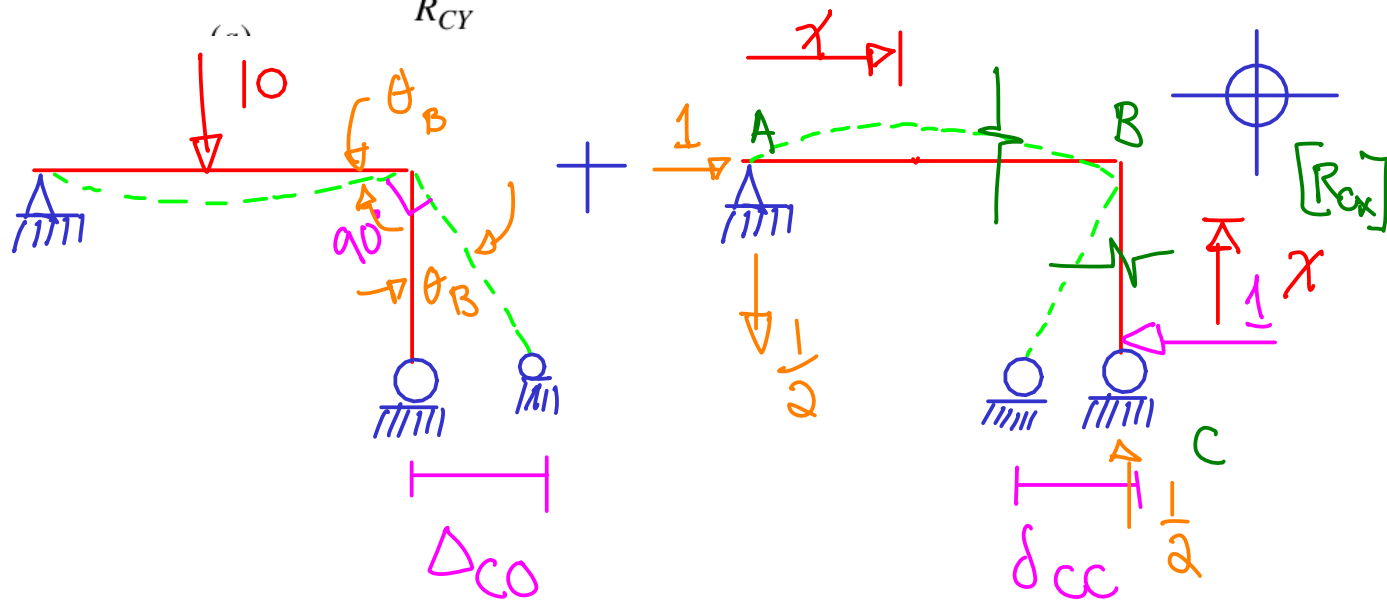


Reacciones

V
M



$$\theta_B = \frac{Pl^2}{16EI}$$

$$\Delta_{CO} = \frac{Pl^2}{16EI} (6) = \frac{10(12)^2(6)}{16EI} = \frac{540}{EI} \rightarrow$$

$$1 \delta_{CC} = \int \frac{M_p M_a dx}{EI} = \int_0^{12} \frac{x}{2} \frac{x}{2} dx + \int_0^6 x x dx$$

Segmento	Origen	Límites	M_p	M_a
AB	A	0-12	$\frac{1}{2}x$	$\frac{1}{2}x$
BC	C	0-6	x	x

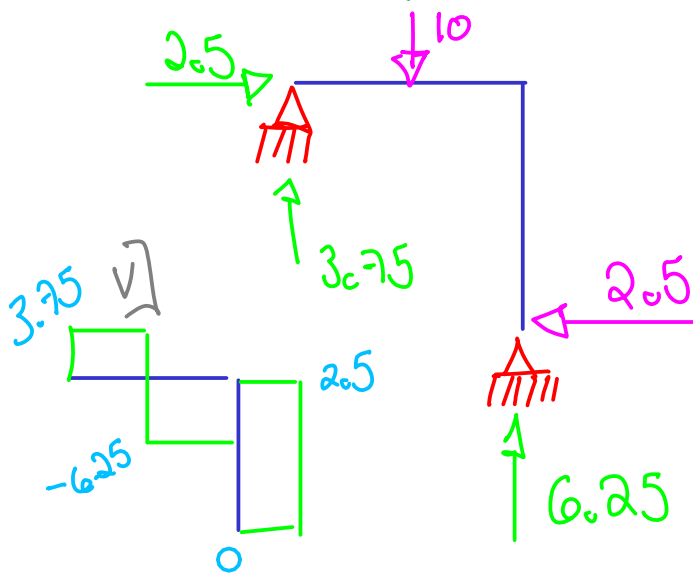
$$\int_0^{12} \frac{x^2}{4} dx + \int_0^6 x^2 dx = \frac{x^3}{12} \Big|_0^{12} + \frac{x^3}{3} \Big|_0^6 = 144 + 72$$

$$\delta_{cc} = \frac{216}{EI}$$

Ecuación de compatibilidad

$$\Delta_{c0} + \delta_{cc} R_{cx} = 0$$

$$\frac{540}{EI} - \frac{216}{EI} R_{cx} = 0 \rightarrow R_{cx} = 2.5$$

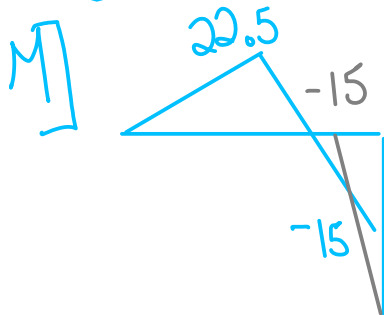


$$\sum M_A = 10(6) + 2.5(6) - R_{cy}(12) = 0$$

$$R_{cy} = 6.25$$

$$\sum F_y = -10 + 6.25 + R_{Ay} = 0$$

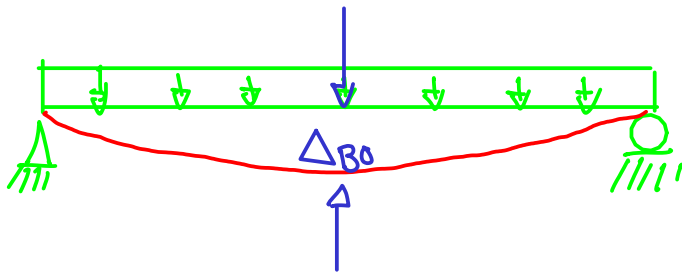
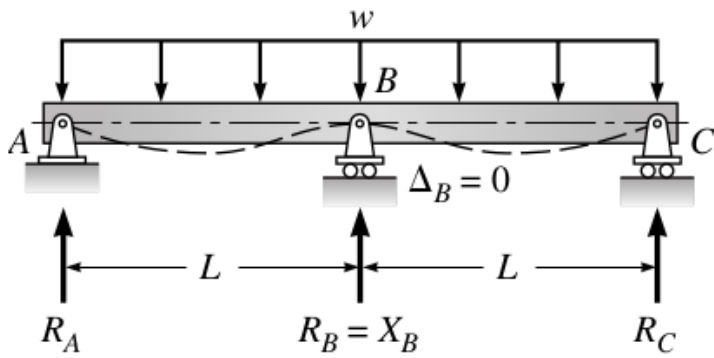
$$R_{Ay} = 3.75$$



$$3.75(6) = 22.5$$

$$22.5 - 6.25(6) = -15$$

$$-15 + 2.5(6) = 0$$



$$\Delta_{B0} = -\frac{5wl^4}{384EI}$$

$$\Delta_{B0} = \frac{-5w(2l)^4}{384EI}$$

Ec. de compatibilidad

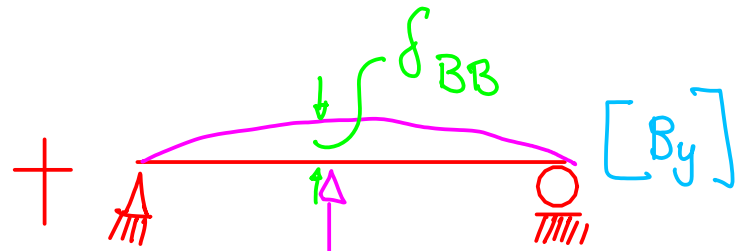
$$\Delta_{B0} + \delta_{BB} B_y = 0$$

$$-\frac{80wl^4}{384EI} + \frac{8l^3}{48EI} B_y = 0$$

$$-\frac{5wl^4}{24EI} + \frac{l^3}{6EI} B_y = 0$$

Reacciones

V
M

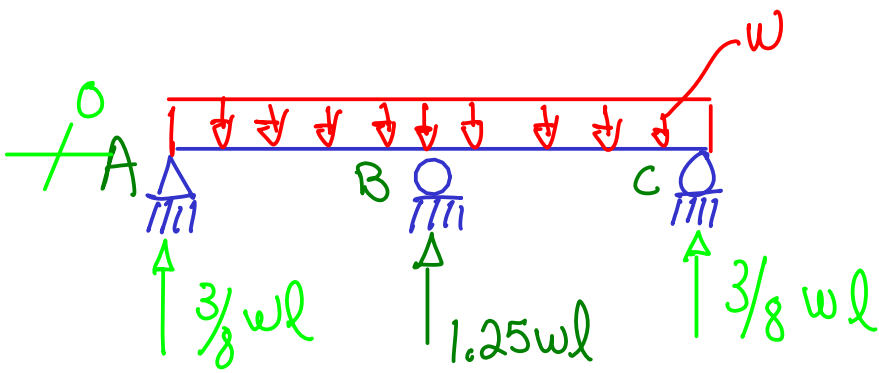


$$\delta_{BB} = \frac{Pl^3}{48EI}$$

$$\delta_{BB} = \frac{1(2l)^3}{48EI}$$

$$\frac{l^3}{6EI} B_y = \frac{5wl^4}{24EI}$$

$$B_y = 1.25wl$$



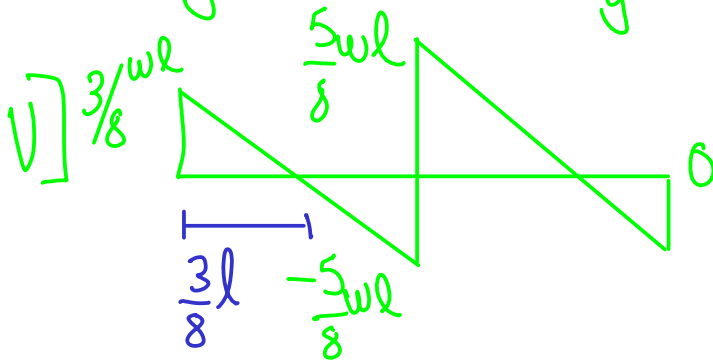
$$\sum M_A = w(2l)(l) - 1.25wl \cdot l - C(2l) = 0$$

$$2wl^2 - 1.25wl^2 = 2cl$$

$$\frac{0.75wl^2}{2l} = c$$

$$c = \frac{3}{8} wl$$

$$\sum F_y = -w(2l) + A_y + 1.25wl + \frac{3}{8}wl = 0 \rightarrow A_y = \frac{3}{8}wl$$



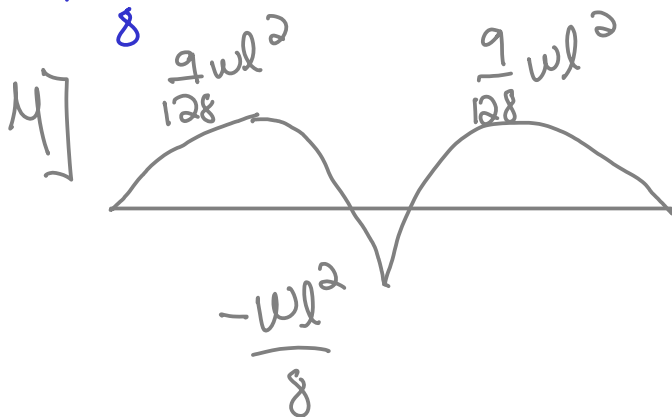
$$\frac{3}{8}wl - wl = -\frac{5}{8}wl$$

$$-\frac{5}{8}wl + \frac{10}{8}wl = \frac{5}{8}wl$$

$$\frac{5}{8}wl - wl = -\frac{3}{8}wl$$

$$\frac{3}{8}wl - w(x) = 0$$

$$x = \frac{3}{8}l$$



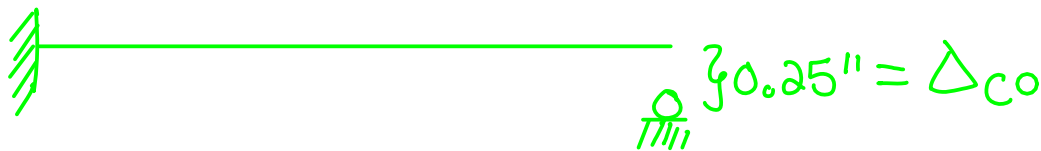
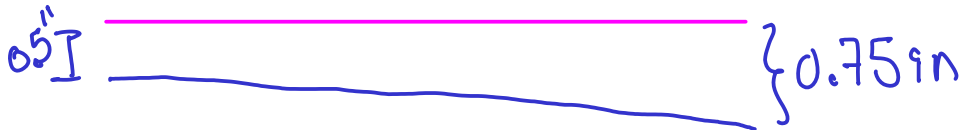
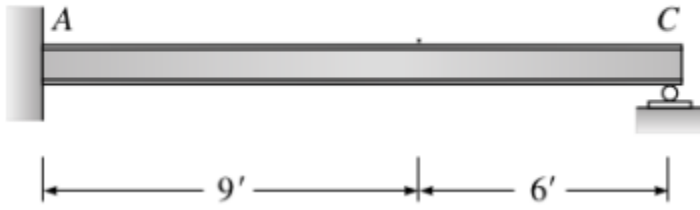
$$\frac{1}{2} \left(\frac{3}{8}l \right) \left(\frac{3}{8}wl \right) = \frac{9}{128}wl^2$$

$$\frac{9}{128}wl^2 - \frac{1}{2} \left(\frac{5}{8}l \right) \left(\frac{5}{8}wl \right) = -\frac{1}{8}wl^2$$

11.9

Asumiendo que no actúa ninguna carga, calcule las reacciones y dibuje el diagrama de fuerza cortante y momento flexionante para la viga mostrada, si el apoyo A se asienta 0.5 in, y el apoyo C se asienta 0.75 in.

Sea $E = 29,000$ ksi & $I = 150$ in⁴.



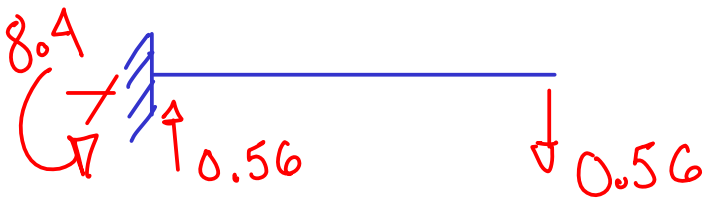
$$\delta_{CC} = \frac{Pl^3}{3EI} = \frac{(1)(15)^3(1728)}{3(29,000)(150)} = 0.447 \text{ in}$$

Ec. compatibilidad

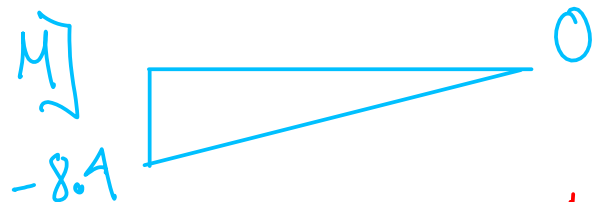
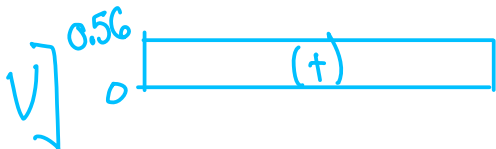
$$-\Delta_{C0} + \delta_{CC} C = 0$$

$$-0.25 + 0.447 C = 0$$

$$C = 0.56 \text{ Kips.}$$

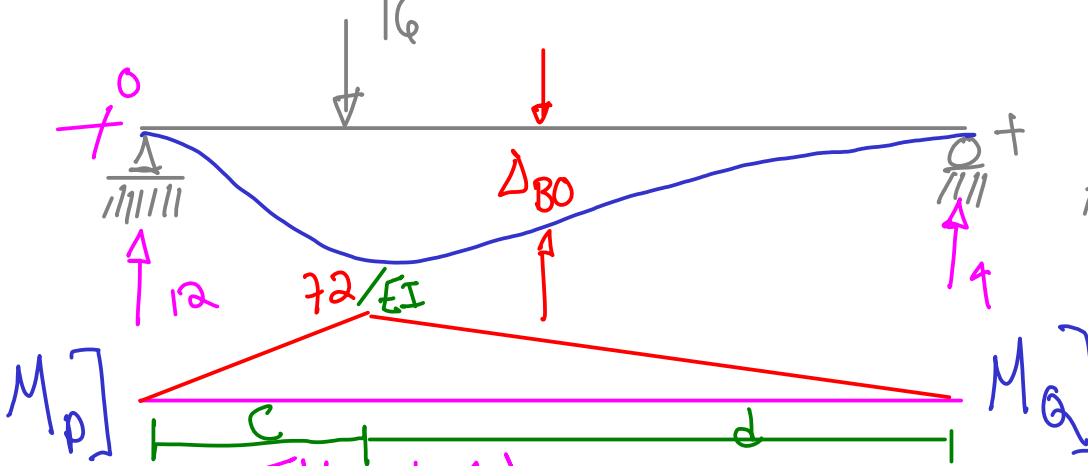
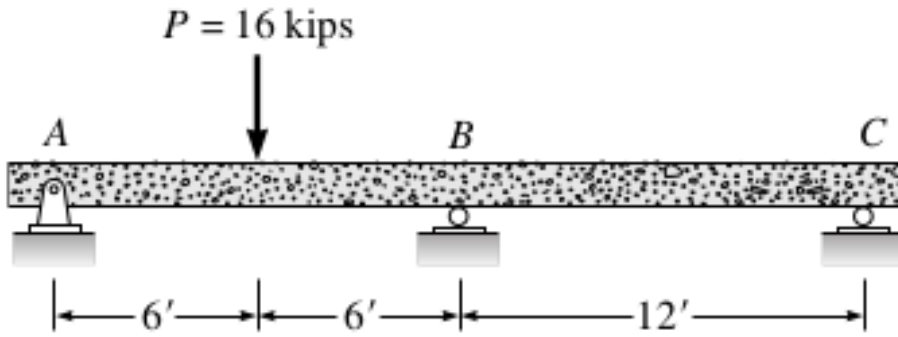


$$(0.56)(15) = 8.4$$



(A) Calcule las reacciones para la viga de la figura asumiendo que los apoyos no se mueven. $EI = \text{cte}$.

(B) Repita los cálculos si el apoyo C se mueve hacia arriba una distancia de $288/EI$ cuando se aplica la carga.



$$\sum M_A = 16(6) - C_y(24) = 0$$

$$C_y = 4$$

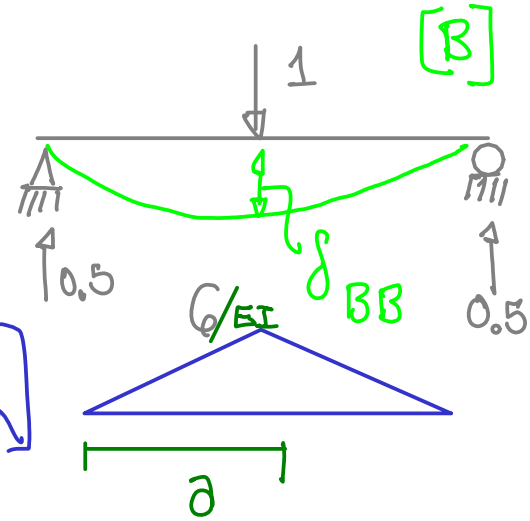
$$12 \times 6 = 72$$

$$4 \times 18 = 72$$

Tabla de integrales

$$\left\{ \begin{aligned} M_p M_Q &= \left(\frac{1}{3} - \frac{(a-c)^2}{6ad} \right) M_1 M_3 L = \left(\frac{1}{3} - \frac{(12-6)^2}{6(12)(18)} \right) (6)(72)(24) \\ \Delta_{B0} &= \frac{3168}{EI} \downarrow \end{aligned} \right.$$

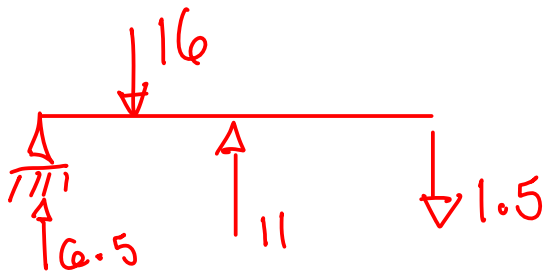
$$\delta_{BB} = \frac{Pl^3}{48EI} = \frac{1(24)^3}{48EI} = \frac{288}{EI} \uparrow$$



Ecuación de Compatibilidad

$$\Delta_{B0} + \delta_{BB} B = 0$$

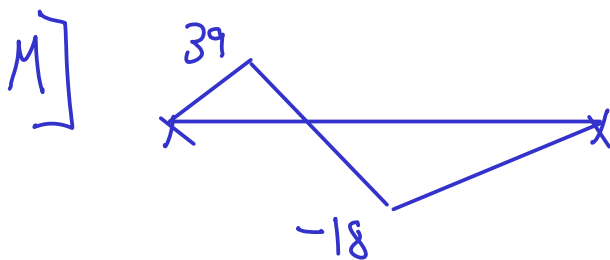
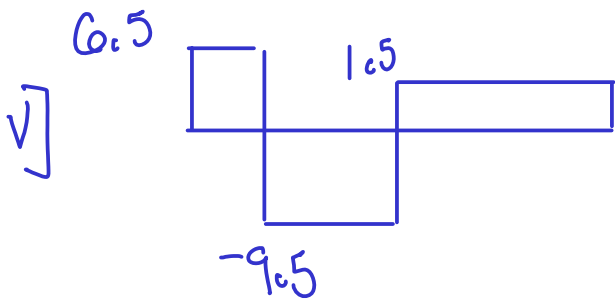
$$-\frac{3168}{EI} + \frac{288}{EI} B = 0 \rightarrow B = 11 \text{ K} \uparrow$$



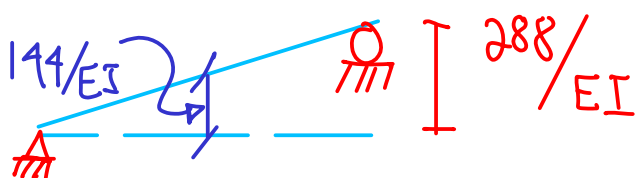
$$\sum M_A = 16(6) - 11(12) - C(24) = 0$$

$$C = 1.5 \text{ K} \downarrow$$

$$\sum F_y = -16 + 11 - 1.5 + A_y = 0 \rightarrow A_y = 6.5 \text{ K}$$



(b)

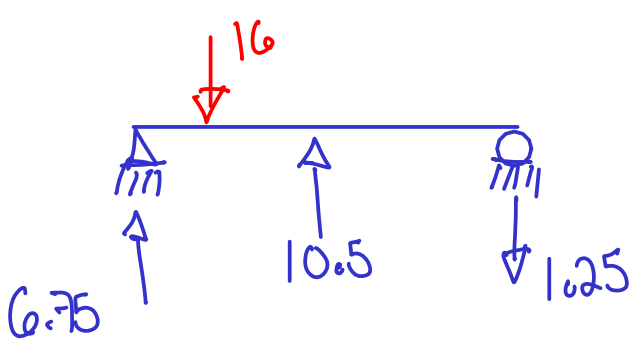


Ecu. de Compatibilidad

$$\Delta_{B0} + \delta_{BB} B = -\frac{144}{EI}$$

$$-\frac{3168}{EI} + \frac{288}{EI} B = -\frac{144}{EI}$$

$$B = 10.5 \text{ K}$$

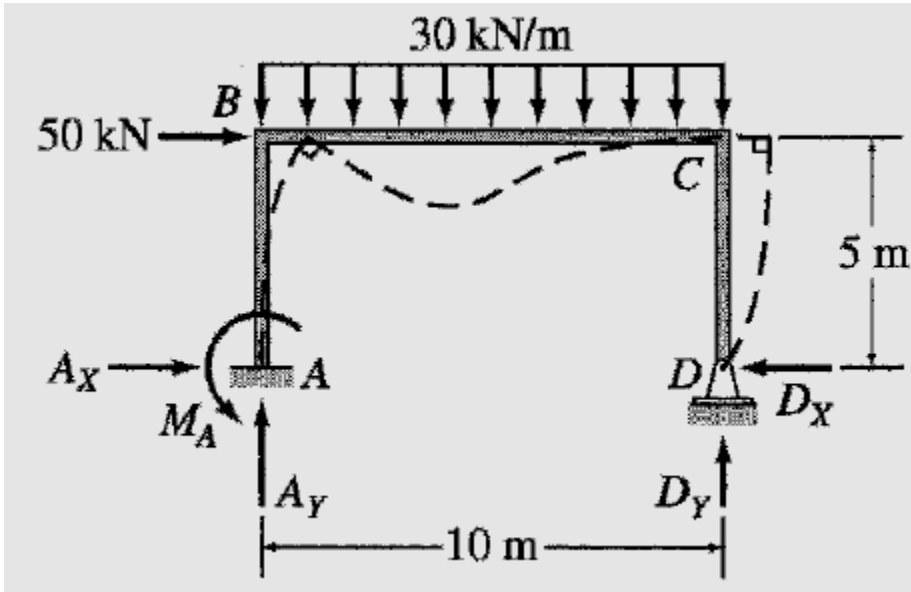


$$\sum M_A = 16(6) - 10.5(12) + C(24) = 0$$

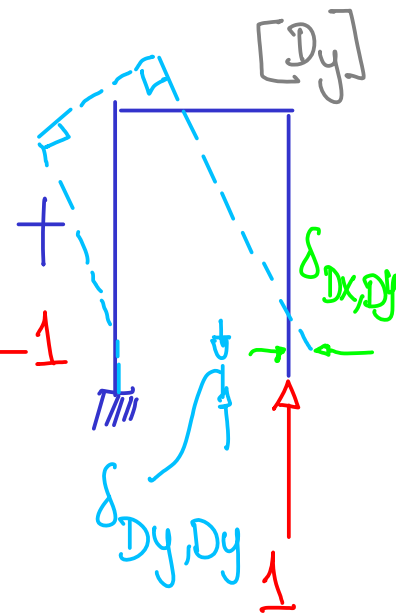
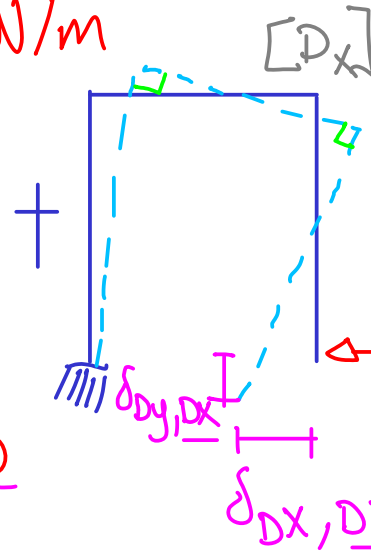
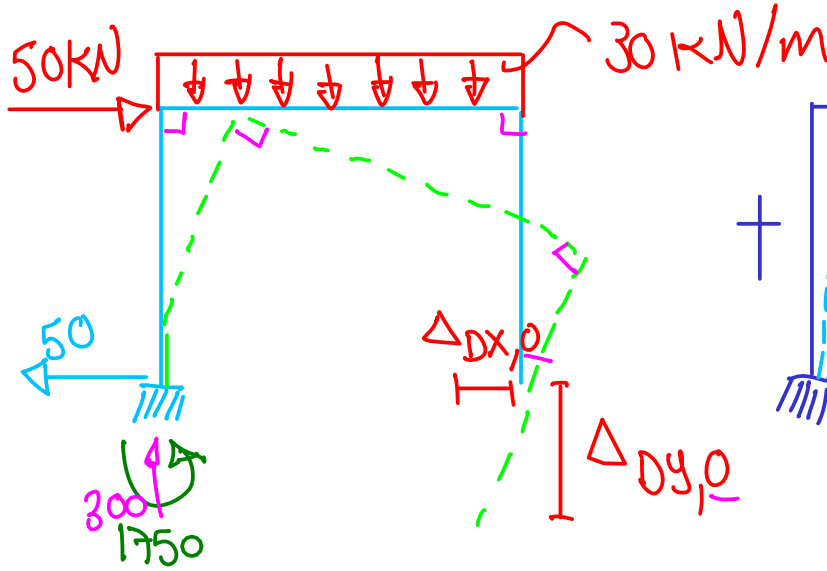
$$C = 1.25 \downarrow$$

$$\sum F_y = A_y - 16 + 10.5 - 1.25 = 0$$

$$A_y = 6.75 \text{ K} \uparrow$$

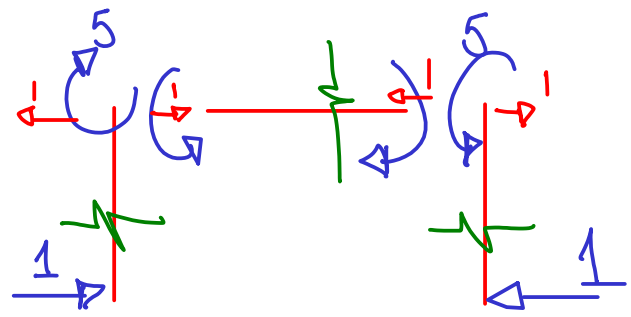
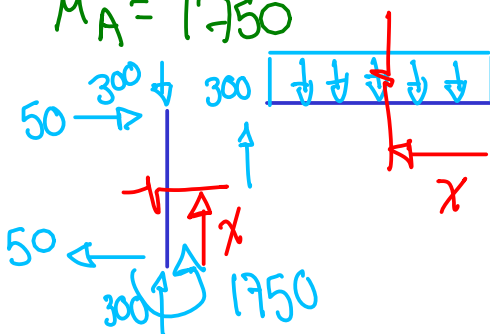


Grado de Hiperestaticidad = 2

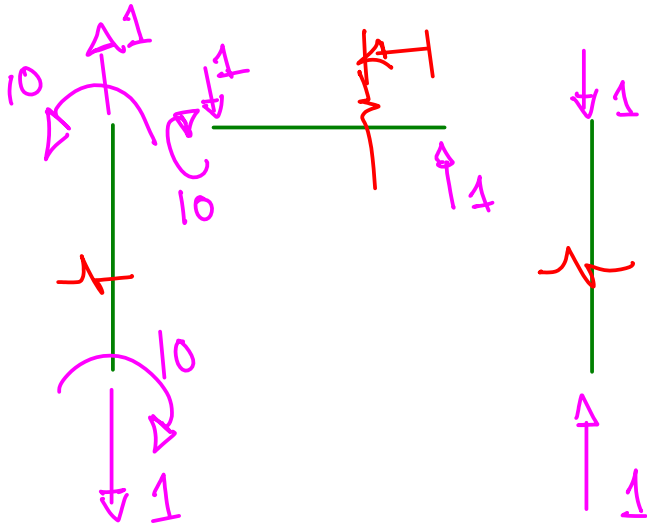


$$\sum M_A = 50(5) + 30(10)(5) + M_A = 0$$

$$M_A = 1750$$



Miembro	Origen	Límites	M_0	M_{Dx}	M_{Dy}
AB	A	0-5	$-1750 + 50x$	$-x$	10
BC	C	0-10	$-15x^2$	-5	x
CD	D	0-5	0	x	0



Ecuaciones de Compatibilidad

$$\Delta_{Dx,0} + \delta_{Dx,Dx} Dx + \delta_{Dx,Dy} Dy = 0$$

$$\Delta_{Dy,0} + \delta_{Dy,Dx} Dx + \delta_{Dy,Dy} Dy = 0$$

$$EI \Delta_{Dx,0} = \int M_0 M_{Dx} dx = \int_0^5 (-1750 + 50x)(-x) dx + \int_0^{10} (-15x^2)(-5) dx$$

$$\int_0^5 1750x - 50x^2 dx + \int_0^{10} 75x^2 dx = \left(\frac{1750x^2}{2} - \frac{50x^3}{3} \right) \Big|_0^5 + \frac{75x^3}{3} \Big|_0^{10}$$

$$= 19,791.67 + 25,000 \rightarrow \Delta_{Dx,0} = \frac{44,791.67}{EI}$$

$$EI \Delta_{Dy,0} = \int (-1750 + 50x)(10) dx + \int_0^{10} (-15x^2)(x) dx$$

$$= \int_0^5 -17500 + 500x dx + \int_0^{10} -15x^3 dx = \left(-17,500x + \frac{500x^2}{2} \right) \Big|_0^5 + \left(-15x^4/4 \right) \Big|_0^{10}$$

$$= -81,250 + -37,500 \rightarrow \Delta_{Dy,0} = \frac{-118,750}{EI}$$

$$EI \delta_{Dx Dx} = \sum \int m_{Dx} m_{Dx} dx = \int_0^5 (-x)(-x) dx + \int_0^{10} (-5)(-5) dx + \int_0^5 (x)(x) dx$$

$$= \frac{x^3}{3} \Big|_0^5 + 25x \Big|_0^{10} + \frac{x^3}{3} \Big|_0^5 \Rightarrow \delta_{Dx Dx} = \frac{333.33}{EI}$$

$$EI \delta_{Dx Dy} = EI \delta_{Dy Dx} = \sum \int M_{Dx} M_{Dy} dx$$

$$= \int_0^5 (-x)(10) dx + \int_0^{10} (-5)(x) dx + \int_0^5 (x)(0) dx$$

$$= -\frac{10x^2}{2} \Big|_0^5 + \frac{-5x^2}{2} \Big|_0^{10} + 0 = -125 - 250 \Rightarrow \frac{-375}{EI}$$

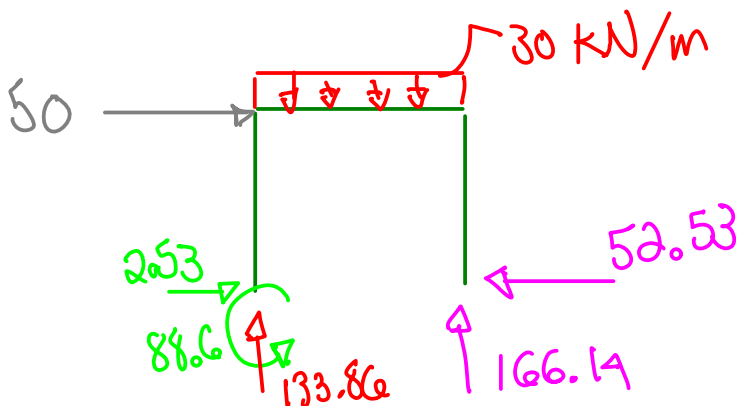
$$EI \delta_{Dy Dy} = \sum \int M_{Dy} M_{Dy} dx = \int_0^5 (10)(10) dx + \int_0^{10} (x)(x) dx$$

$$= 100x \Big|_0^5 + \frac{x^3}{3} \Big|_0^{10} = 500 + 333.33 \Rightarrow \frac{833.33}{EI}$$

$$44,791.67 + 333.33 D_x - 375 D_y = 0$$

$$-118,750 - 375 D_x + 833.33 D_y = 0$$

$$\begin{bmatrix} 333.33 & -375 \\ -375 & 833.33 \end{bmatrix}^{-1} \begin{bmatrix} -44791.67 \\ 118750 \end{bmatrix} = \begin{bmatrix} 52.5342 \\ 166.1411 \end{bmatrix} = \begin{matrix} D_x \leftarrow \\ D_y \uparrow \end{matrix}$$



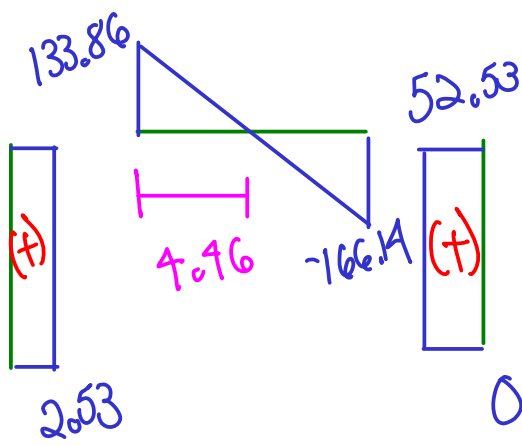
$$\sum M_A = 50(5) + 30(10)(5) - 166.14(10) + M_A = 0$$

$$M_A = -88.6 \therefore 88.6 \curvearrowright$$

$$\sum F_y = -30(10) + 166.14 + A_y = 0$$

$$A_y = 133.86$$

V

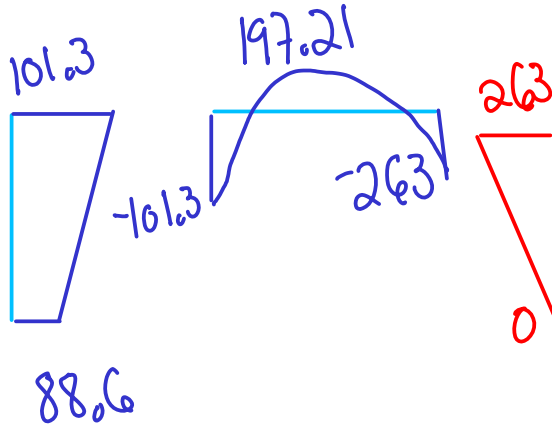


$$133.86 - 30(10) = -166.14$$

$$133.86 - 30x = 0$$

$$x = 4.46 \text{ m}$$

M



$$88.6 + 2.53(5) = 101.3$$

$$-101.3 + \frac{1}{2}(4.46)(133.86)$$

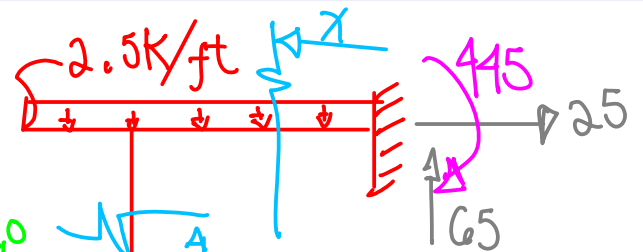
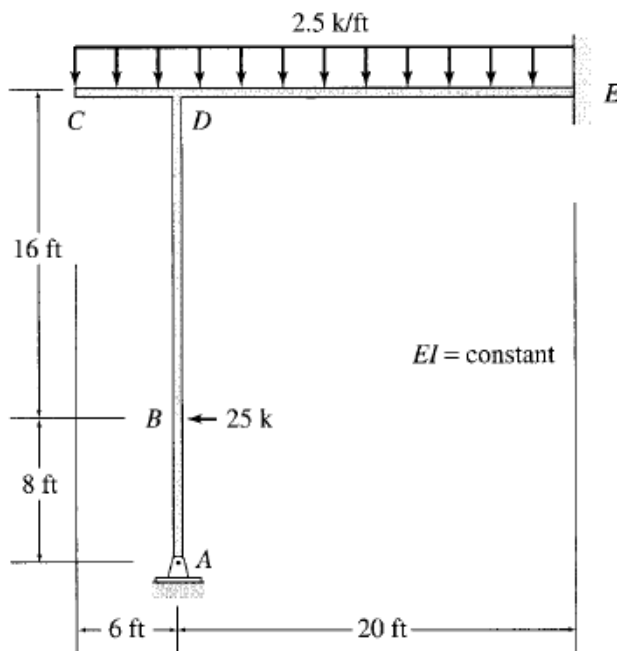
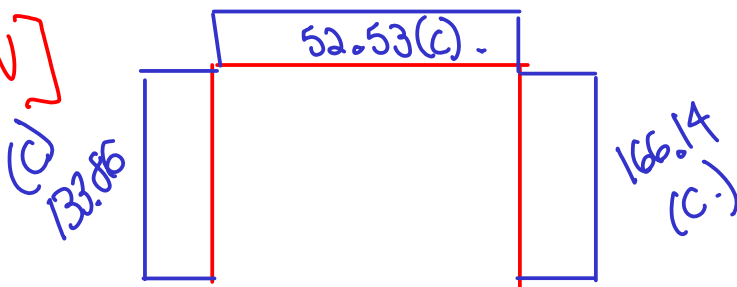
$$= 197.21$$

$$197.21 - \frac{1}{2}(10-4.46)(166.14)$$

$$= -263$$

$$-263 + 52.53(5) = 0 \checkmark$$

N



$$\Delta A_x, 0$$

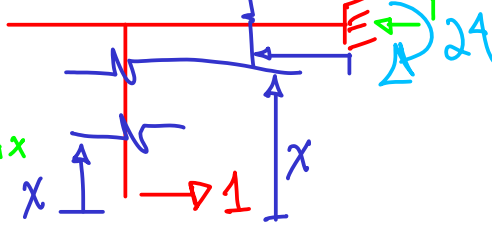
$$\Delta A_y, 0$$

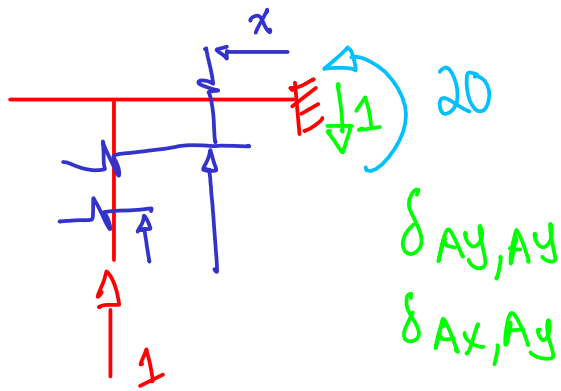
$$25 \text{ k}$$

$$\sum M_E = M_E + 2.5(20)(13) - 25(16) = 0 \rightarrow M_E = -145$$

$$\delta_{A_x, A_x}$$

$$\delta_{A_y, A_x}$$





Segmento	Origen	Limites	M_0	M_{Ax}	M_{Ay}
AB	A	0-8	0	$-x$	0
BD	A	8-24	$25(x-8)$	$-x$	0
DE	E	0-20	$-2.5x(\frac{x}{2}) + 65x - 445$	-24	$-x + 20$

$$\Delta_{Ax,0} EI = \int M_0 M_{Ax} dx = \int_0^8 (0)(x) dx + \int_8^{24} (25x-200)(-x) dx + \int_0^{20} (-1.25x^2 + 65x - 445)(-24) dx$$

$$\int_8^{24} (25x-200)(-x) dx = -59733.3333$$

$$\int_0^{20} (-1.25x^2 + 65x - 445)(-24) dx = -18400$$

$$-59733.3333 + -18400 = \frac{-78133.3333}{EI} = \Delta_{Ax,0}$$