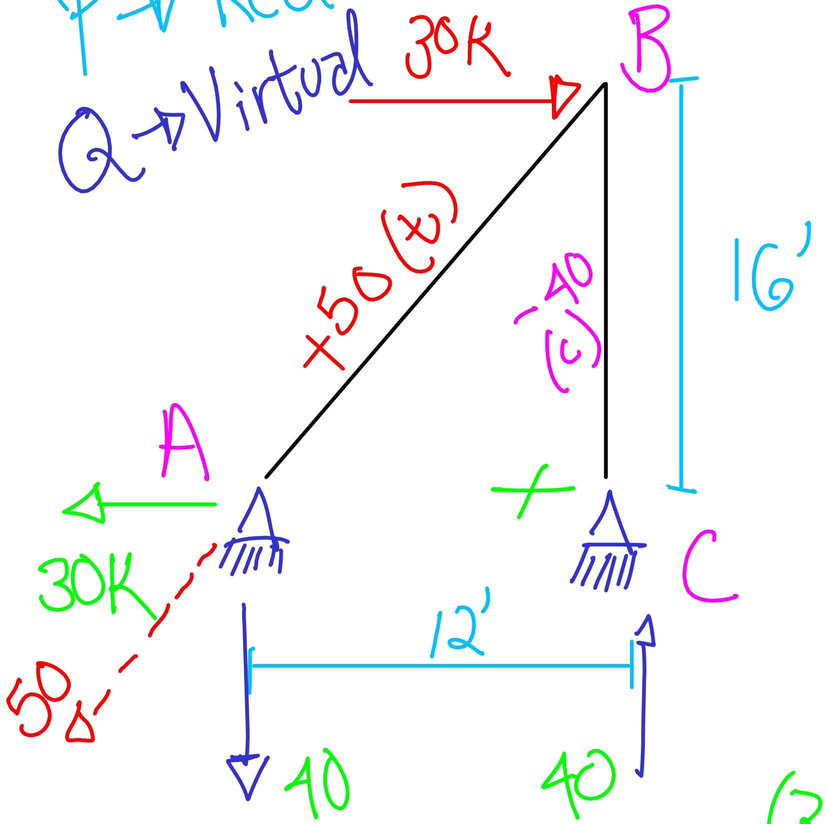
P → Real

Q → Virtual



Determinar el desplazamiento horizontal y vertical en B.

$E = 30{,}000 \text{ ksi}$

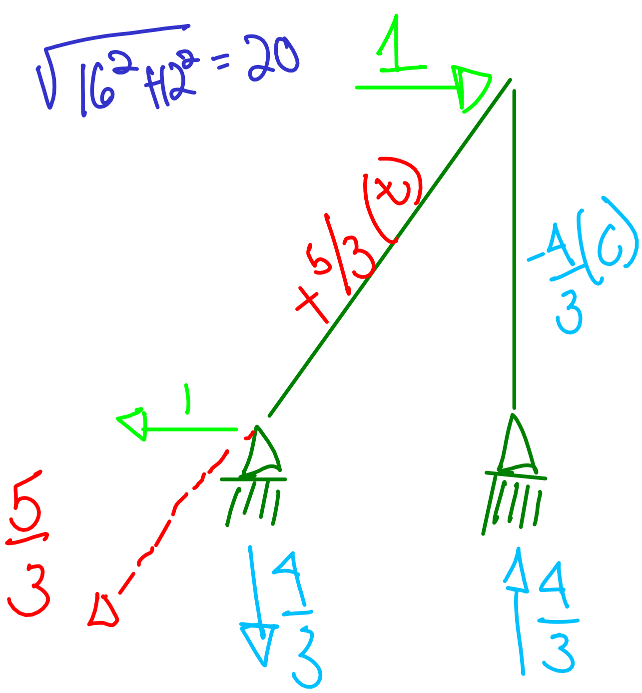
$A = 2 \text{ in}^2$

$$\sum Q \delta_p = \sum F_Q \frac{F_p L}{AE}$$

$$(30)(16) = x(12) \rightarrow x = 40$$

$$\sqrt{30^2 + 40^2} = 50$$

$$\sqrt{16^2 + 12^2} = 20$$

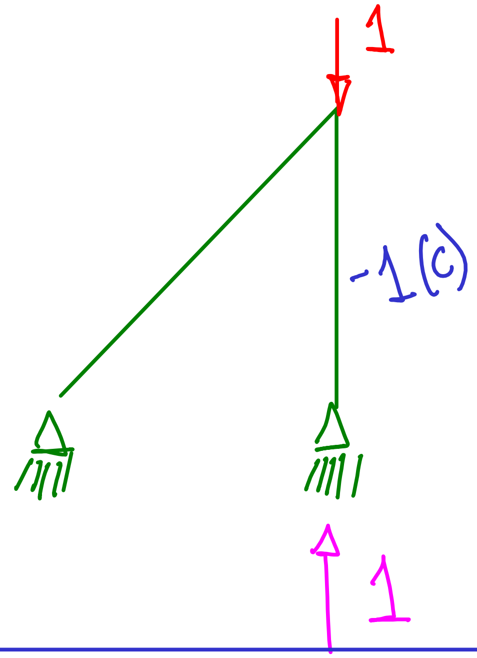


$$(1)(16) = x(12) \rightarrow x = \frac{4}{3}$$

$$\sqrt{\left(\frac{4}{3}\right)^2 + 1^2} = \frac{5}{3}$$

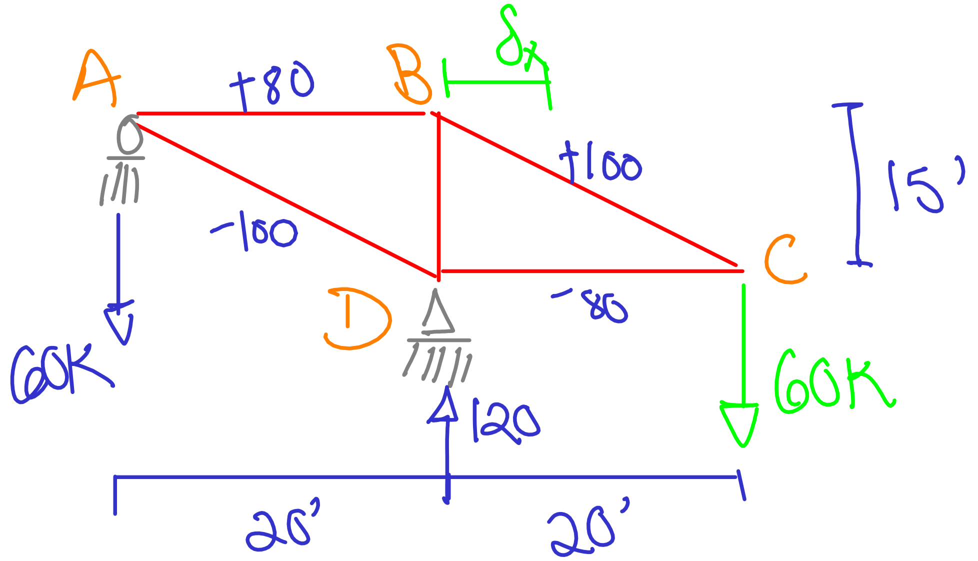
$$1\,\delta_p = \frac{5}{3}\left(\frac{50 \times 20 \times 12}{2 \times 30{,}000}\right) - \frac{4}{3}\left(\frac{-40 \times 16 \times 12}{2 \times 30{,}000}\right)$$

$$\delta_p = 0.5 \text{ in} \rightarrow$$



$$1\delta_p = -1\left(\frac{-40\times16\times12}{2\times30{,}000}\right)+0$$

$$\delta_p = 0.128 \text{ in} \downarrow$$

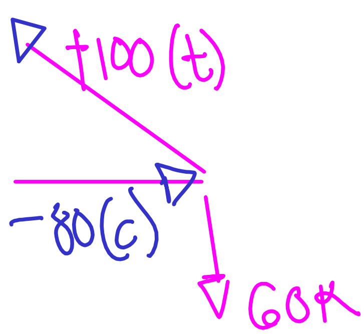


E = 30,000 Ksi

A_{AD} & $A_{BC} = 5 \text{ in}^2$

Demás áreas $= 4 \text{ in}^2$

Nodo C

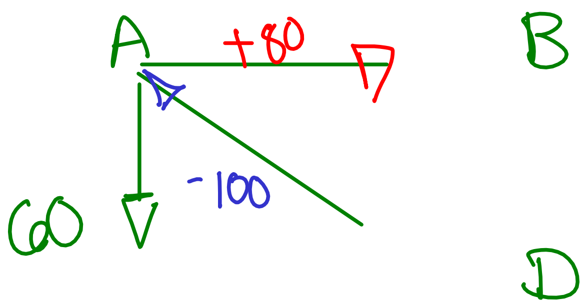


$$\Sigma F_y = -60 + F_{BC}\left(\frac{15}{25}\right) = 0 \rightarrow F_{BC} = 100T$$

$$\Sigma F_x = -100\left(\frac{20}{25}\right) + F_{DC} = 0$$

$$F_{DC} = -80 \rightarrow 80(C)$$

Nodo A



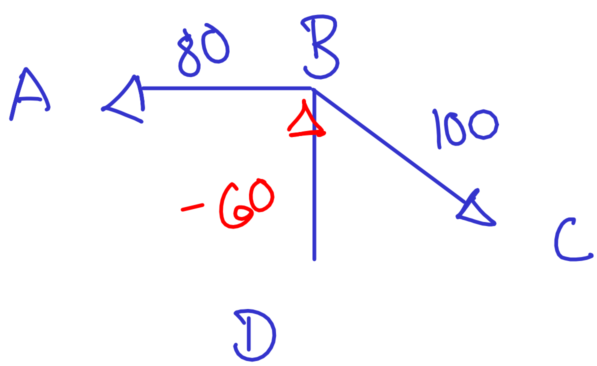
$$\Sigma F_y = -60 + F_{AD}\left(\frac{15}{25}\right) = 0$$

$$F_{AD} = 100\ (c)$$

$$\Sigma F_x = -100\left(\frac{20}{25}\right) + F_{AB} = 0$$

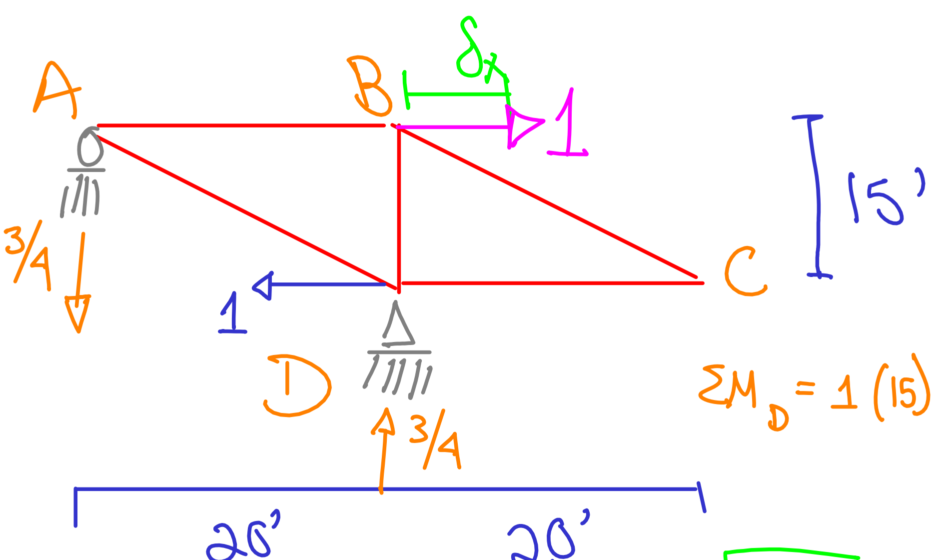
$$F_{AB} = 80\ T$$

Nodo B



$$\Sigma F_y = -100\left(\frac{15}{25}\right) + F_{BD} = 0$$

$$F_{BD} = 60\ (c)$$

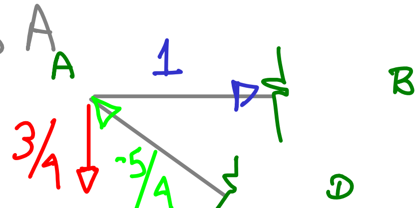


$$\Sigma M_D = 1(15) - x(20) = 0$$

$$x = \frac{15}{20} = \frac{3}{4}$$

$$\sqrt{15^2 + 20^2} = 25$$

Nudo A



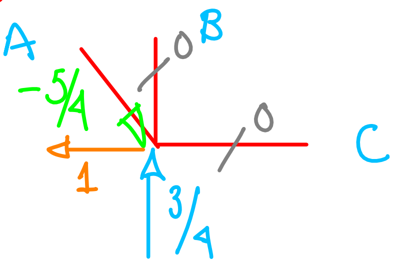
$$\Sigma F_y = -\frac{3}{4} + F_{AD}\left(\frac{15}{25}\right) = 0$$

$$F_{AD} = 5/4\ (c)$$

$$\Sigma F_x = -\frac{5}{4}\left(\frac{20}{25}\right) + F_{AB} = 0$$

$$\hookrightarrow F_{AB} = \underline{1}\ (T)$$

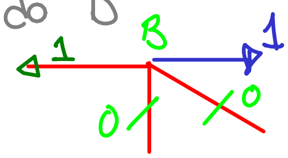
Nodo D



$$\frac{5}{4}\left(\frac{20}{25}\right) = 1$$

$$\frac{5}{4}\left(\frac{15}{25}\right) = \frac{3}{4}$$

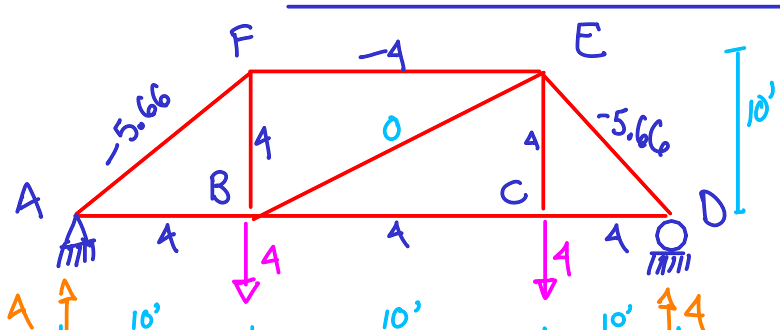
Nodo B



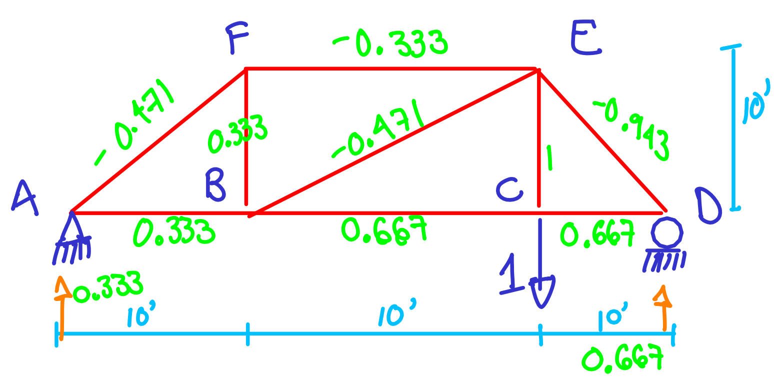
Miembro	F_Q	F_p	L	A	F_pF_QL/A
AB	+1	+80	20	4	+400
BC	0	+100	25	5	0
AD	-5/4	-100	25	5	+625
BD	0	-60	15	4	0
DC	0	-80	20	4	0
					1025

$$\Sigma F_Q \frac{F_p L}{AE} = \Sigma Q\, \delta_p$$

$$\frac{1025}{30{,}000}(12) = 1\,\delta_p \qquad \underline{\underline{\delta_p = 0.41\ \text{in}}}$$



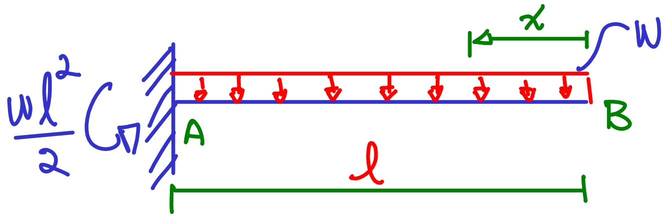
Determine el desplazamiento vertical de la junta C de la armadura de acero. El área de la sección transversal de cada miembro es de A = 0.5 in², y E = 29,000 ksi.



Miembro	F_Q	F_p	L	A	F_pF_QL/A
AB	0.333	4	10	0.5	26.64
BC	0.667	4	10	0.5	53.36
CD	0.667	4	10	0.5	53.36
DE	-0.943	-5.66	14.14	0.5	160.94
FE	-0.333	-4	10	0.5	26.64
EB	-0.471	0	14.14	0.5	0
BF	0.333	4	10	0.5	26.64
AF	-0.471	-5.66	14.14	0.5	75.39
CE	1	4	10	0.5	80
					493

$$\Sigma F_Q \frac{F_p L}{AE} = \Sigma Q\,\delta_p$$

$$\frac{493}{29{,}000}(12) = 1\,\delta_p \qquad \underline{\underline{\delta_p = 0.204 \text{ in}}}$$

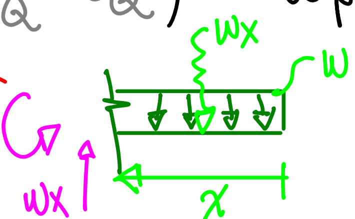


Utilizando el trabajo virtual, calcule la deflexión y la rotación en B para la viga con carga uniformemente repartida. (Viga en voladizo, viga en cantilever).

Viga real

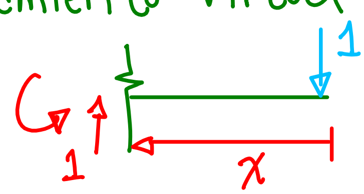
$$W_Q = U_Q;\ \Sigma Q\delta_p = \int M_Q \frac{M_p\,dx}{EI}$$

M_p: $wx\left(\frac{x}{2}\right) = \frac{wx^2}{2}$



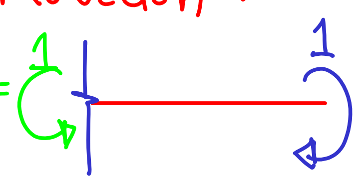
Desplazamiento Virtual.

$M_Q = x$



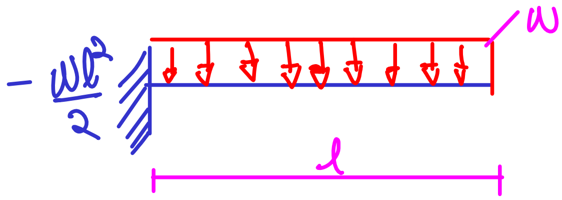
Rotacion Virtual

$M_Q = 1$



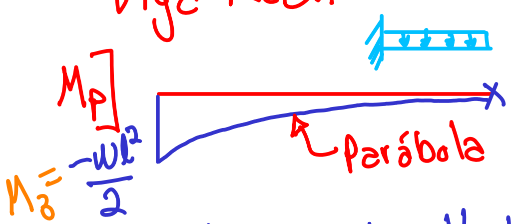
$$EI\,1\,\delta_p = \int_0^l (x)\left(\frac{wx^2}{2}\right)dx = \frac{wx^4}{8}\Big|_0^l = \frac{wl^4}{8} \rightarrow \delta_B = \frac{wl^4}{8EI}\downarrow$$

$$EI\,\theta_p = \int_0^\ell (1)\left(\frac{wx^2}{2}\right)dx = \frac{wx^3}{6}\Big|_0^\ell = \frac{w\ell^3}{6} \to \theta_B = \frac{w\ell^3}{6EI}$$

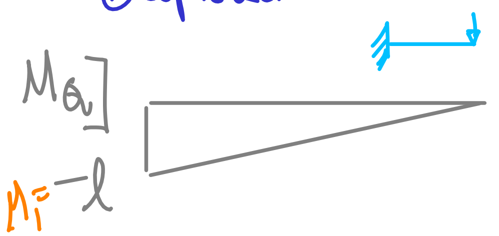


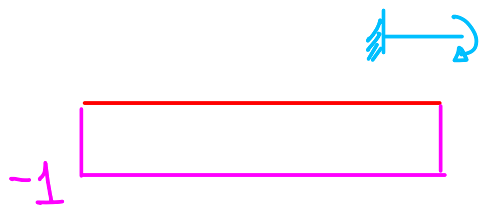
Lo mismo pero con las áreas de los diagramas y las tablas de productos de integrales.

Viga Real.



Desplazamiento Virtual. Rotación Virtual.





Del Libro $\to \int_0^\ell M_Q M_p\,dx$

$$= \frac{1}{4} M_1 M_3 L = \frac{1}{4}(-\ell)\left(\frac{-w\ell^2}{2}\right)\ell = \frac{w\ell^4}{8}$$

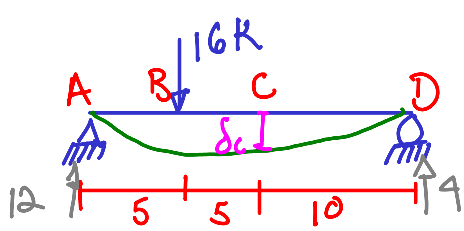
$$1\,\delta_p = \int_0^L \frac{M_Q M_p}{EI}\,dx \to 1\,EI\,\delta_p = \frac{w\ell^4}{8}$$

$$\delta_B = \frac{w\ell^4}{8EI}$$

Del Libro $\to \int M_Q M_p\,dx = \frac{1}{3} M_1 M_3 L$

$$\frac{1}{3}(-1)\left(-\frac{w\ell^2}{2}\right)\ell = \frac{w\ell^3}{6}$$

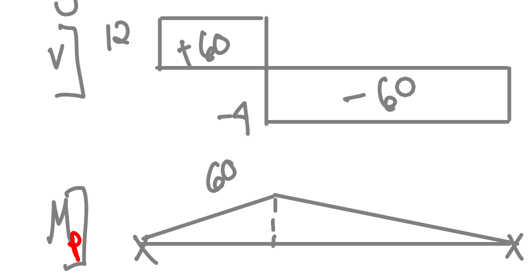
$$EI\,1\,\theta_p = \int M_p M_Q\,dx \to \theta_B = \frac{w\ell^3}{6EI}$$



$E = 29{,}000$ $I = 240\ in^4$

¿δ_c?

Viga Real



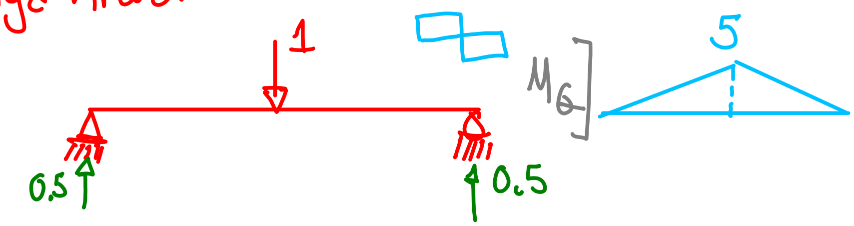
$\Sigma M_A = 16(5) - D_y(20) = 0$

$D_y = 4\,k$

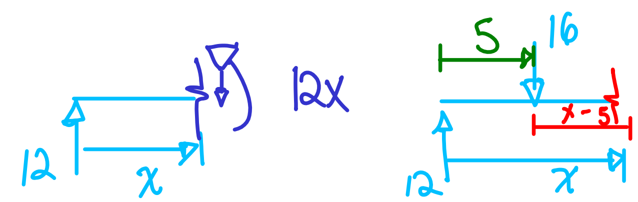
$\Sigma F_y = 16 - 4 - A_y = 0$

$A_y = 12\,k$

Viga Virtual



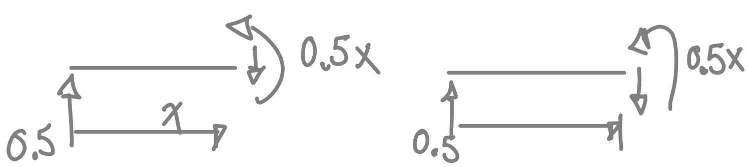
Segmento	Origen	Rango	M_Q	M_p
AB	A	0-5	0.5x	12x
BC	A	5-10	0.5x	-4x+80
DC	D	0-10	0.5x	4x

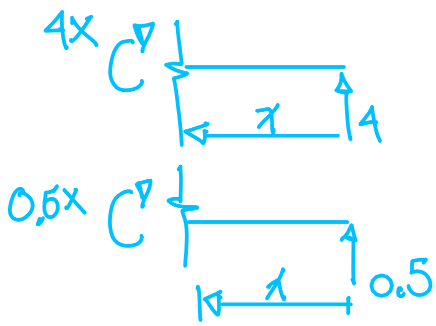


$M = 12x - 16(x-5)$

$M = 12x - 16x + 80$

$M = -4x + 80$





$$EI\,1\,Kip\,\delta_{P_C} = \int_0^5 (12x)(0.5x)dx + \int_5^{10} (-4x+80)(0.5x)\,dx + \int_0^{10} (4x)(0.5x)\,dx$$

$$\frac{6x^3}{3}\Big|_0^5 + \int_5^{10} -2x^2 + 40x\,dx + \frac{2x^3}{3}\Big|_0^{10}$$

$$250 + \left. -\frac{2x^3}{3} + \frac{40x^2}{2} \right|_5^{10} + 666.67 = 250 + 1333.33 + 666.67 - 416.66 = 1833.33$$