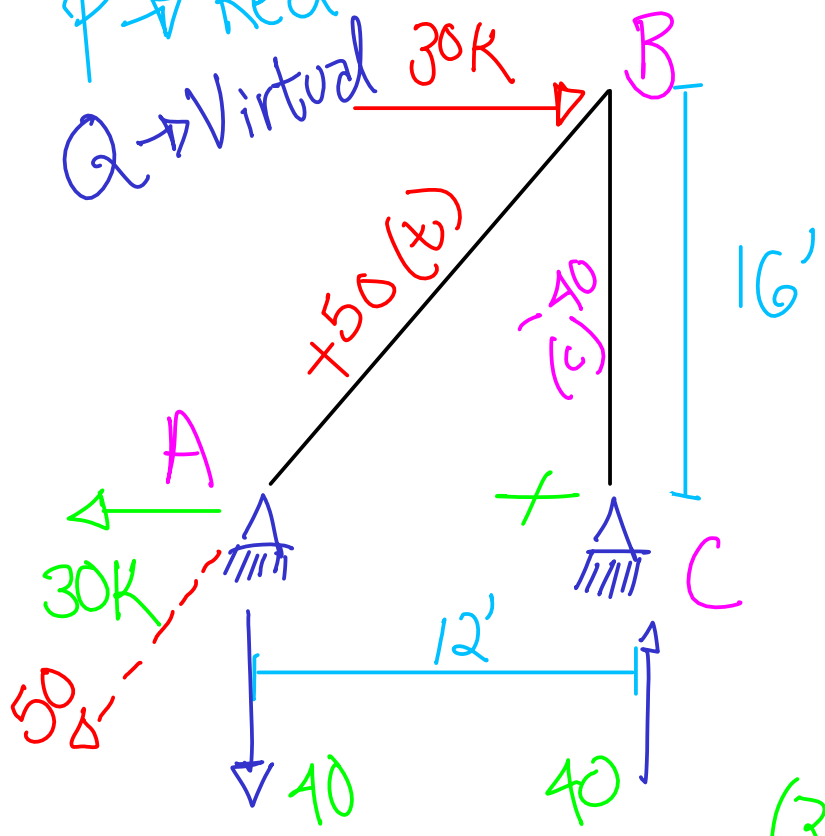


P → Real
Q → Virtual



Determinar el desplazamiento horizontal y vertical en B.

$$E = 30,000 \text{ Ksi}$$

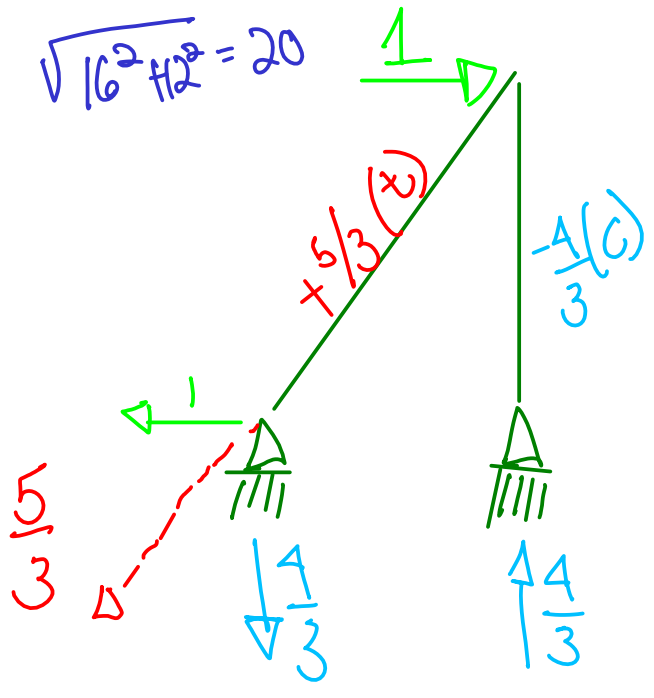
$$A = 2 \text{ in}^2$$

$$\sum Q \delta_p = \sum F_a \frac{F_p L}{AE}$$

$$(30)(16) = \chi(12) \rightarrow \chi = 40$$

$$\sqrt{30^2 + 40^2} = 50$$

$$\sqrt{16^2 + 12^2} = 20$$



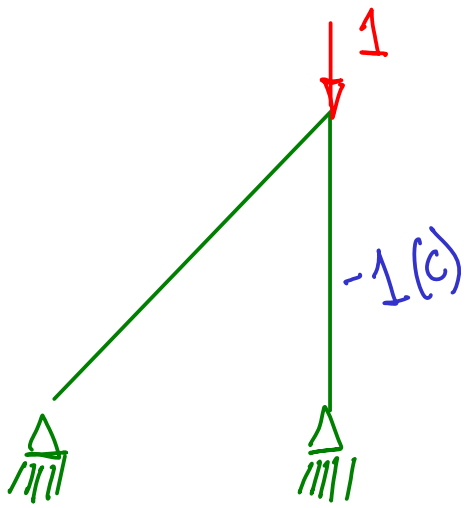
$$(1)(16) = \chi(12) \rightarrow \chi = \frac{4}{3}$$

$$\sqrt{\left(\frac{4}{3}\right)^2 + 12^2} = \frac{5}{3}$$

$$\delta_p = \frac{5}{3} \left(\frac{50 \times 20 \times 12}{2 \times 30,000} \right)$$

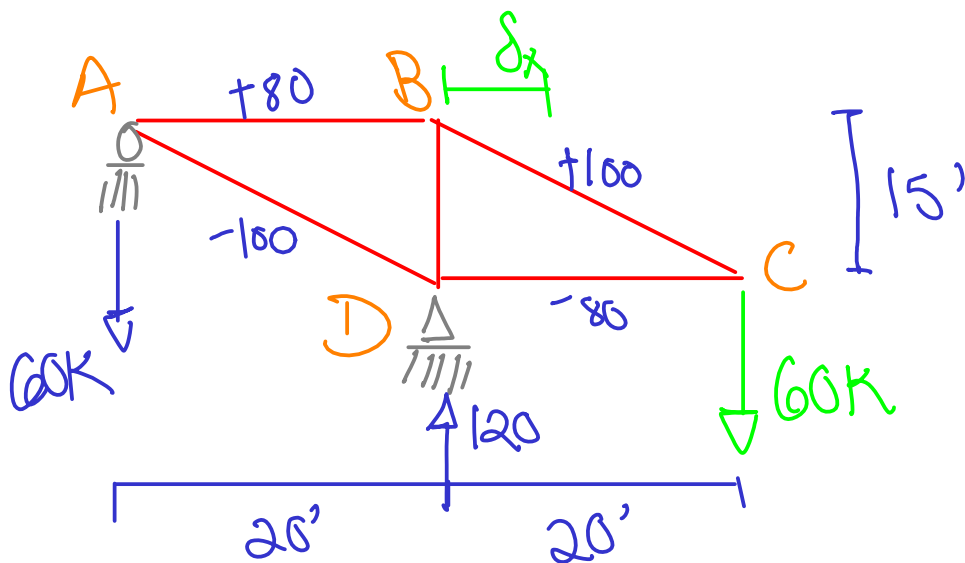
$$- \frac{4}{3} \left(\frac{-40 \times 16 \times 12}{2 \times 30,000} \right)$$

$$\delta_p = 0.5 \text{ in} \rightarrow$$



$$\Delta \delta_p = -1 \left(\frac{-10 \times 16 \times 12}{2 \times 30,000} \right) + 0$$

$$\delta_p = 0.128 \text{ in } \downarrow$$

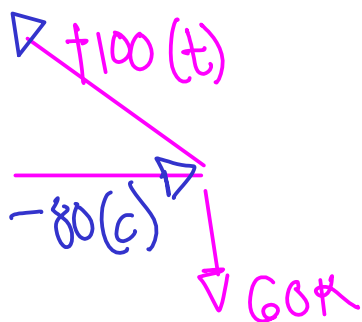


$$E = 30,000 \text{ Ksi}$$

$$A_{AD} \& A_{BC} = 5 \text{ in}^2$$

$$\text{Demás áreas} = 4 \text{ in}^2$$

Nodo C

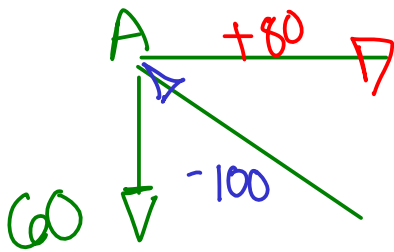


$$\sum F_y = -60 + F_{BC} \left(\frac{15}{25} \right) = 0 \rightarrow F_{BC} = 100 \text{ T}$$

$$\sum F_x = -100 \left(\frac{20}{25} \right) + F_{DC} = 0$$

$$F_{DC} = -80 \rightarrow 80(c)$$

Nodo A



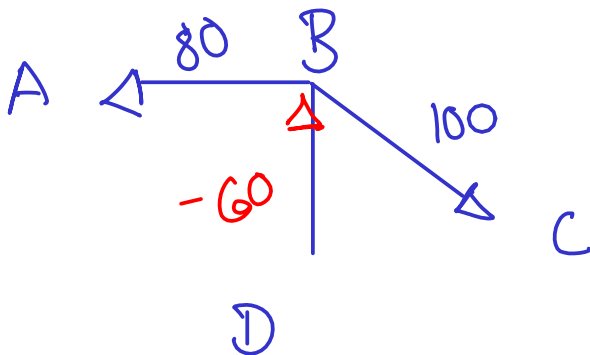
B $\Sigma F_y = -60 + F_{AD} \left(\frac{15}{25} \right) = 0$

$F_{AD} = 100 \text{ (c)}$

D $\Sigma F_x = -100 \left(\frac{20}{25} \right) + F_{AB} = 0$

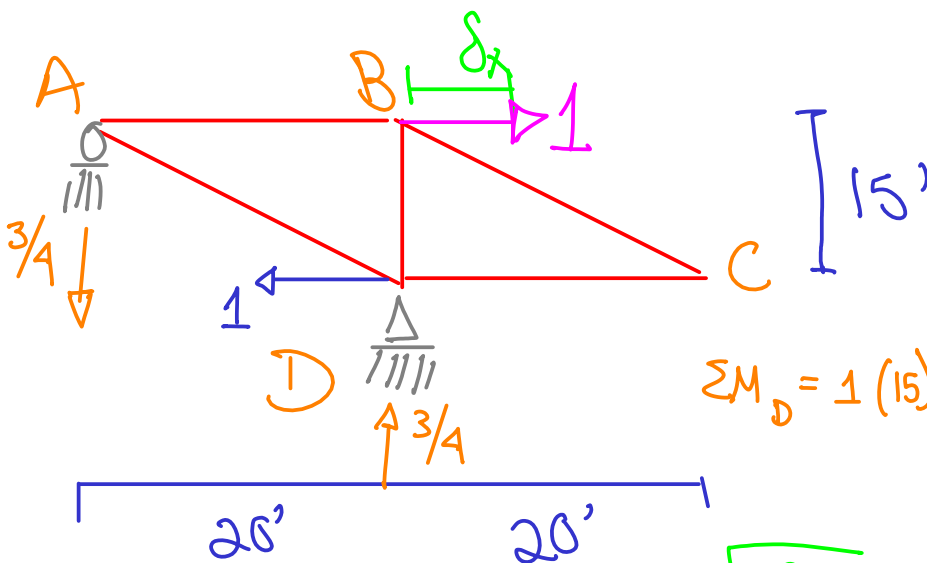
$F_{AB} = 80 \text{ T}$

Nodo B



$\Sigma F_y = -100 \left(\frac{15}{25} \right) + F_{BD} = 0$

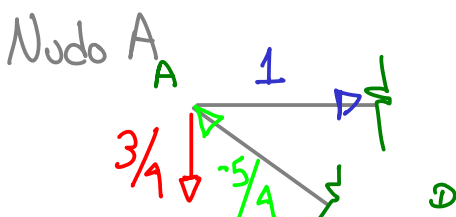
$F_{BD} = 60 \text{ (c)}$



$\Sigma M_D = 1(15) - \chi(20) \Rightarrow$

$\chi = \frac{15}{20} = \frac{3}{4} \checkmark$

$\sqrt{15^2 + 20^2} = 25$



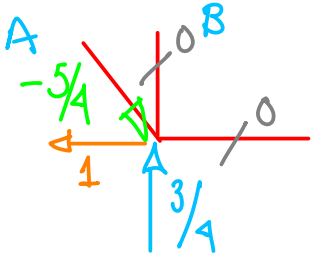
B $\Sigma F_y = -\frac{3}{4} + F_{AD} \left(\frac{15}{25} \right) = 0$

$F_{AD} = \frac{5}{4} \text{ (c)}$

$$\Sigma F_x = -\frac{5}{4} \left(\frac{20}{25} \right) + F_{AB} = 0$$

$$\hookrightarrow F_{AB} = 1 \text{ (T)}$$

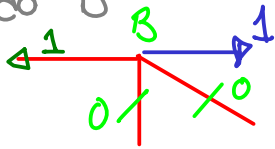
Nodo D



$$\frac{5(20)}{4(25)} = 1$$

$$C \quad \frac{5(15)}{4(25)} = \frac{3}{4}$$

Nodo B

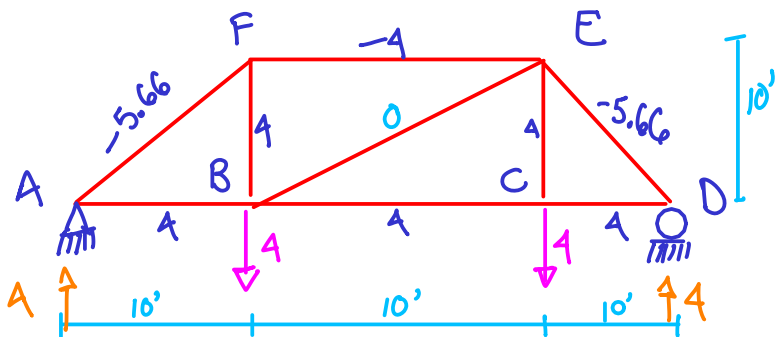


Miembro	F_Q	F_P	L	A	$F_P F_Q L/A$
AB	+1	+80	20	4	+400
BC	0	+100	25	5	0
AD	-5/4	-100	25	5	+625
BD	0	-60	15	4	0
DC	0	-80	20	4	0
					<u>1025</u>

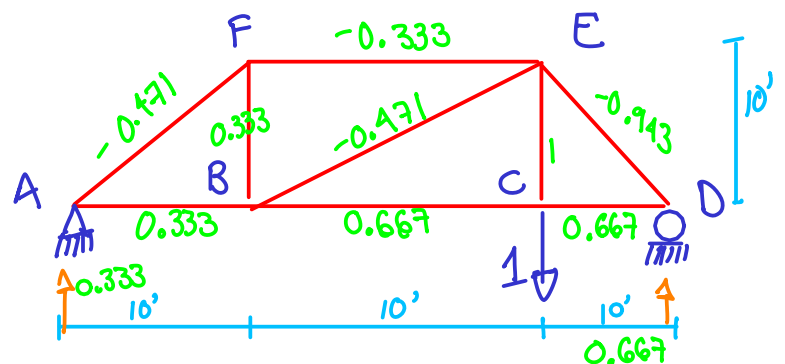
$$\Sigma F_Q \frac{F_P L}{AE} = \Sigma Q \delta_P$$

$$\frac{1025(12)}{30,000} = 1 \delta_P$$

$$\delta_P = 0.41 \text{ in}$$



Determine el desplazamiento vertical de la junta C de la armadura de acero. El área de la sección transversal de cada miembro es de $A = 0.5 \text{ in}^2$, y $E = 29,000 \text{ ksi}$.

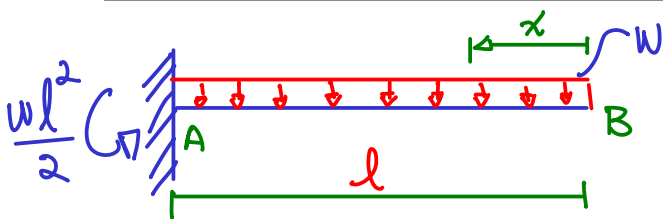


Miembro	F_Q	F_P	L	A	$F_Q F_P L / A$
AB	0.333	4	10	0.5	26.64
BC	0.667	4	10	0.5	53.36
CD	0.667	4	10	0.5	53.36
DE	-0.943	-5.66	14.14	0.5	150.94
FE	-0.333	-4	10	0.5	26.64
EB	-0.471	0	14.14	0.5	0
BF	0.333	4	10	0.5	26.64
AF	-0.471	-5.66	14.14	0.5	75.39
CE	1	4	10	0.5	80
					<hr/> 493

$$\sum F_Q \frac{F_P L}{AE} = \sum Q \delta_P$$

$$\frac{493}{29,000} (12) = 1 \delta_P$$

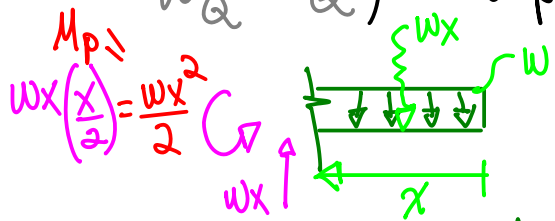
$$\delta_P = \underline{\underline{0.204 \text{ in}}}$$



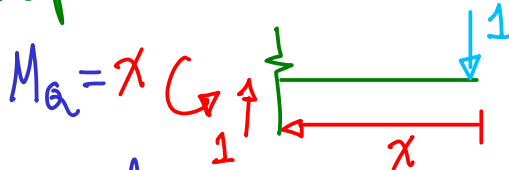
Utilizando el trabajo virtual, calcule la deflexión y la rotación en B para la viga con carga uniformemente repartida. (Viga en voladizo, viga en cantilever).

Viga real

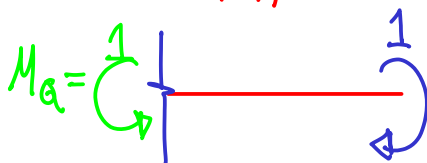
$$W_Q = U_Q; \sum Q \delta_P = \int M_Q \frac{M_P dx}{EI}$$



Desplazamiento Virtual.

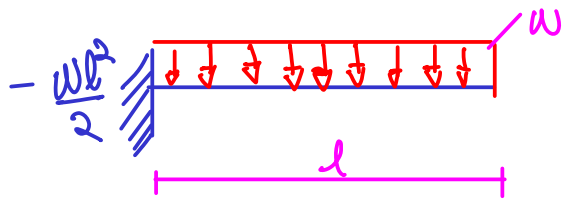


Rotación Virtual



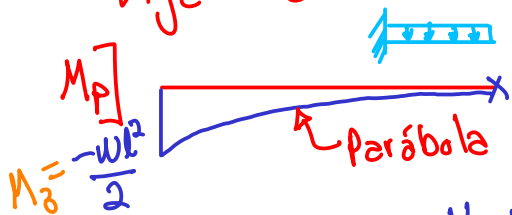
$$EI \delta_P = \int_0^l (x) \left(\frac{wx^2}{2} \right) dx = \frac{wx^4}{8} \Big|_0^l = \frac{wl^4}{8} \rightarrow \delta_B = \frac{wl^4}{8EI} \downarrow$$

$$EI \theta_B = \int_0^l (1) \left(\frac{wx^2}{2} \right) dx = \frac{wx^3}{6} \Big|_0^l = \frac{wl^3}{6} \rightarrow \theta_B = \frac{wl^3}{6EI}$$

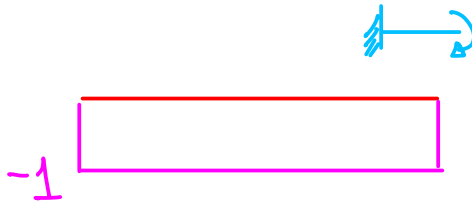
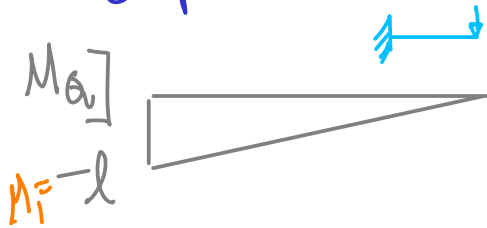


Lo mismo pero con las áreas de los diagramas y las tablas de productos de integrales.

Viga Real.



Desplazamiento Virtual. Rotación Virtual.



Del Libro $\rightarrow \int_0^l M_Q M_P dx$

$$= \frac{1}{4} M_1 M_3 L = \frac{1}{4} (-l) \left(-\frac{wl^2}{2} \right) l = \frac{wl^4}{8}$$

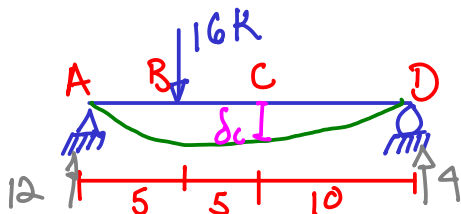
$$1 \delta_P = \int_0^L \frac{M_Q M_P}{EI} dx \rightarrow 1 EI \delta_P = \frac{wl^4}{8}$$

$$\delta_B = \frac{wl^4}{8EI}$$

Del Libro $\rightarrow \int M_Q M_P dx = \frac{1}{3} M_1 M_3 L$

$$\frac{1}{3} (-l) \left(-\frac{wl^2}{2} \right) l = \frac{wl^3}{6}$$

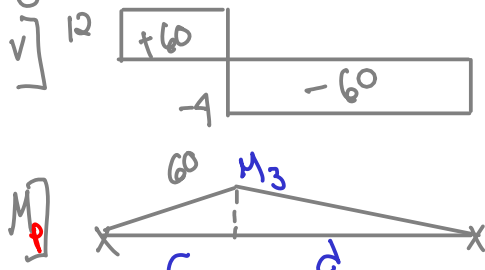
$$EI \theta_B = \int M_P M_Q dx \rightarrow \theta_B = \frac{wl^3}{6EI}$$



$E = 29,000$ $I = 210 \text{ in}^4$

$c \delta_c$?

Viga Real



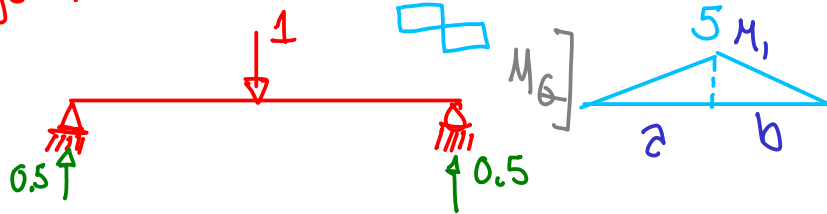
$$\sum M_A = 16(5) - D_y(20) = 0$$

$$D_y = 4 \text{ k}$$

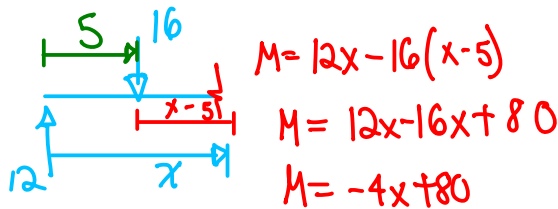
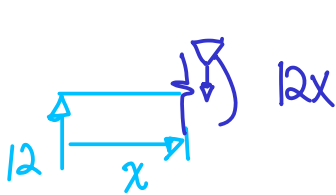
$$\sum F_y = 16 - 4 - A_y = 0$$

$$A_y = 12 \text{ k}$$

Viga Virtual



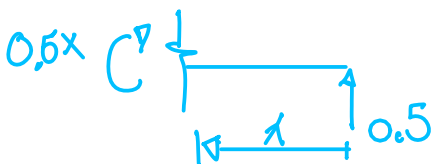
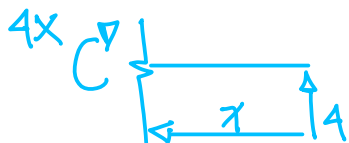
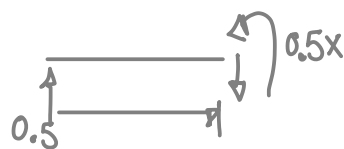
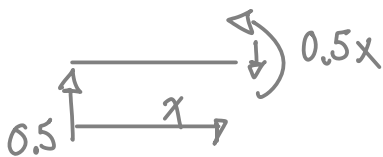
Segmento	Origen	Rango	M_Q	M_p
AB	A	0-5	$0.5x$	$12x$
BC	A	5-10	$0.5x$	$-4x + 80$
DC	D	0-10	$0.5x$	$4x$



$$M = 12x - 16(x-5)$$

$$M = 12x - 16x + 80$$

$$M = -4x + 80$$



$$EI \delta_{PC} = \int_0^5 (12x)(0.5x) dx + \int_5^{10} (-4x+80)(0.5x) dx + \int_0^{10} (4x)(0.5x) dx$$

$$\frac{6x^3}{3} \Big|_0^5 + \int_5^{10} -2x^2 + 40x dx + \frac{2x^3}{3} \Big|_0^{10}$$

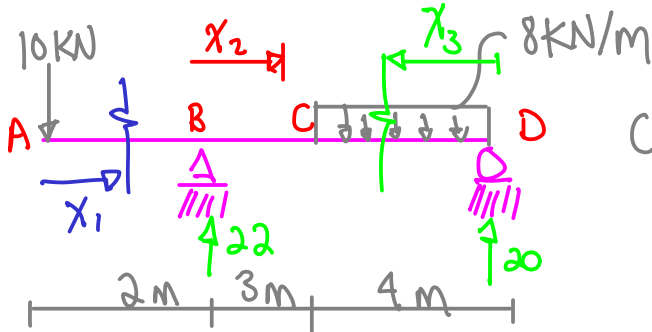
$$250 + \left[\frac{-2x^3}{3} + \frac{40x^2}{2} \right]_5^{10} + 666.67 = 250 + 1333.33 + 666.67 - 416.66 = 1833.33$$

$$EI \delta_{pc} = 1833.33$$

$$\delta_{pc} = \frac{1833.33 (12^3)}{29,000 (240)} = 0.45 \text{ in}$$

Tablas de Integrales

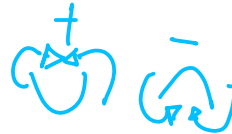
$$\left(\frac{1}{3} - \frac{(a-c)^2}{6ad} \right) M_1 M_3 L = \left(\frac{1}{3} - \frac{(10-5)^2}{6(10)(15)} \right) (5)(60)(20) \left(\frac{12^3}{29,000 \times 240} \right) = 0.45 \text{ in}$$



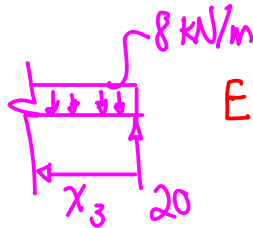
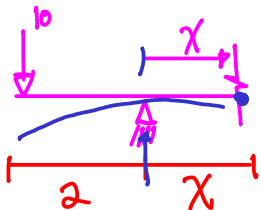
Calcule la deflexión en C; EI = Constante.

$$\sum M_B = 10(2) - 8(4)(5) + D_y(7) = 0 \rightarrow D_y = 20$$

$$\sum F_y = -10 - 8(4) + 20 + B_y = 0 \rightarrow B_y = 22$$



Segmento	Origen	Limites	M_p	M_Q	
AB	A	0-2	$-10x$	0	
BC	B	0-3	$-10(x+2) + 22x$	$(\frac{1}{7})x$	$-10x - 20 + 22x = 12x - 20$
DC	D	0-4	$20x - 8x(\frac{x}{2})$	$(\frac{3}{7})x$	$20x - 4x^2$



$$EI \delta_{pc} = \int_0^2 -10x dx + \int_0^3 (12x - 20)(\frac{1}{7}x) dx + \int_0^4 (20x - 4x^2)(\frac{3}{7}x) dx$$

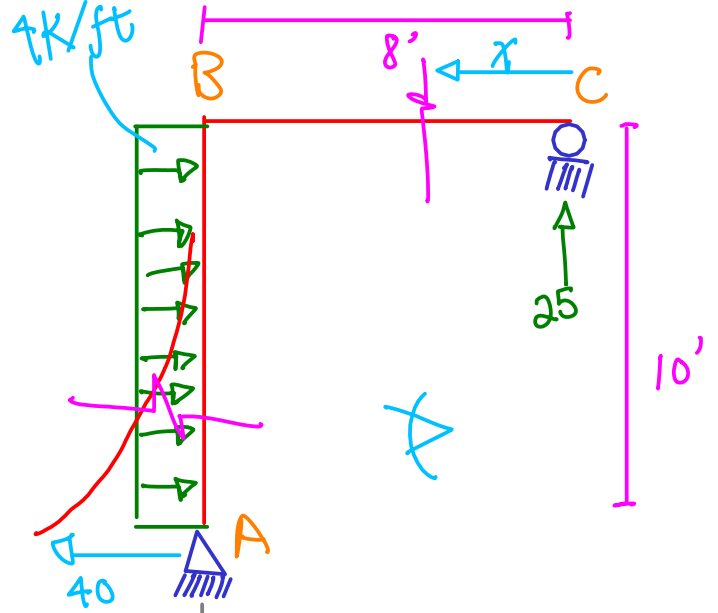
$$\frac{48x^2}{7} - \frac{80x}{7}$$

$$\frac{60x^2}{7} - \frac{12x^3}{7}$$

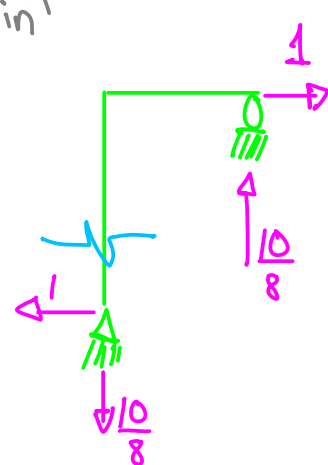
$$EI \delta_{pc} = \left(\frac{48x^3}{21} - \frac{80x^2}{14} \right) \Big|_0^2 + \left(\frac{60x^3}{21} - \frac{12x^4}{28} \right) \Big|_0^3$$

$$= 10.286 + 73.14 = 83.43$$

$$\delta_{pc} = 83.43 / EI \downarrow$$



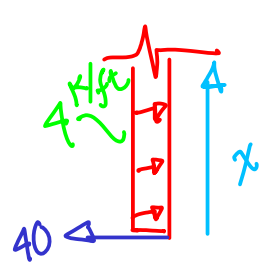
\int Horiz. C
 $E = 29,000 \text{ ksi}$
 $I = 600 \text{ in}^4$



$$\sum M_A = 4(10)(5) - C_y(8) = 0 \rightarrow C_y = 25 \text{ K}$$

$$\sum F_x = 4(10) - A_x = 0 \rightarrow A_x = 40$$

Segmento	Origen	Lmites	M_p	M_q
AB	A	0-10	$40x - 4x\left(\frac{x}{2}\right)$	$1x$
CB	C	0-8	$25x$	$\frac{10}{8}x$



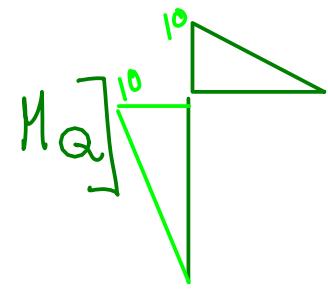
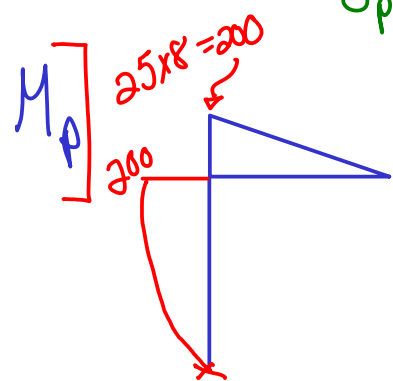
$$\alpha = 40x - 2x^2$$

$$EI \delta_{p_{h_c}} = \int_0^{10} (40x - 2x^2)(x) dx + \int_0^8 (25x)(1.25x) dx$$

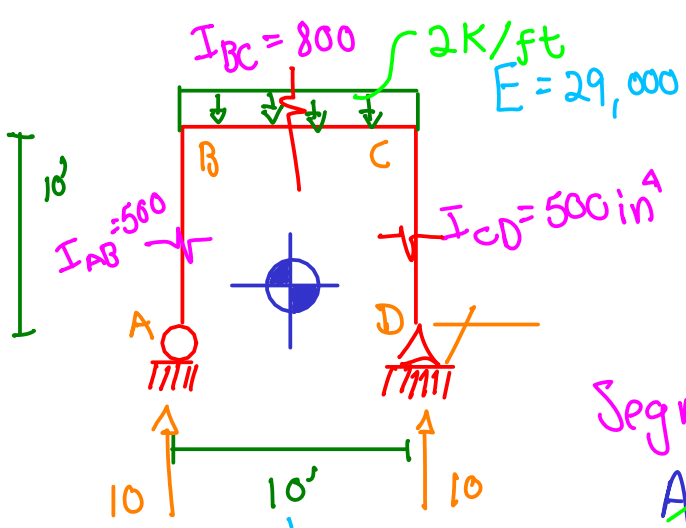
$$\beta = 40x^2 - 2x^3 \quad \gamma = 31.25x^2$$

$$EI \delta_{p_{h_c}} = \left(\frac{40x^3}{3} - \frac{2x^4}{4} \right) \Big|_0^{10} + \frac{31.25x^3}{3} \Big|_0^8 = 8,333.33 + 5,333.33$$

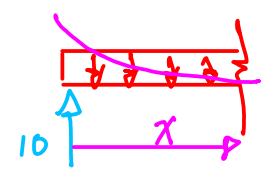
$$\delta_{p_{h_c}} = \frac{13,666.67}{EI} = \frac{13666.67 (12^3)}{29,000 \times 600} = 1.357 \text{ in} \rightarrow$$



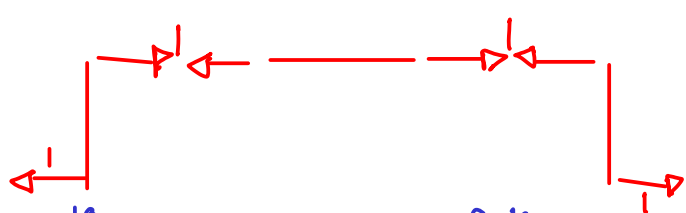
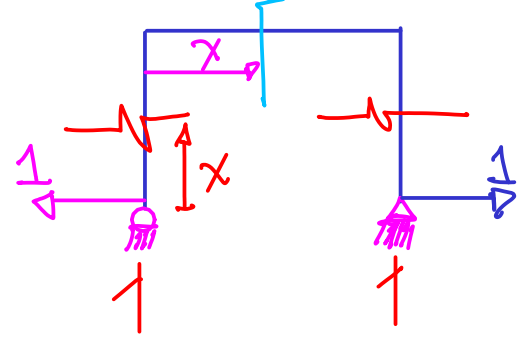
Obtener mediante áreas.



$\delta_{\text{Horiz. A}} = ?$



Segmento	Origen	Limites	M_p	M_q
AB	A	0-10	0	$1x$
BC	B	0-10	$10x - 2x\left(\frac{x}{2}\right)$	10
CD	D	0-10	0	$1x$



$$EI \delta_{h_A} = \int_0^{10} (10x - x^2)(10) dx = \int_0^{10} 100x - 10x^2 dx$$

$$= \left(\frac{100x^2}{2} - \frac{10x^3}{3} \right) \Big|_0^{10} = 1666.67$$

$$\delta_{h_A} = \frac{1666.67 (12^3)}{(29,000)(800)} = 0.124 \text{ in} \leftarrow$$