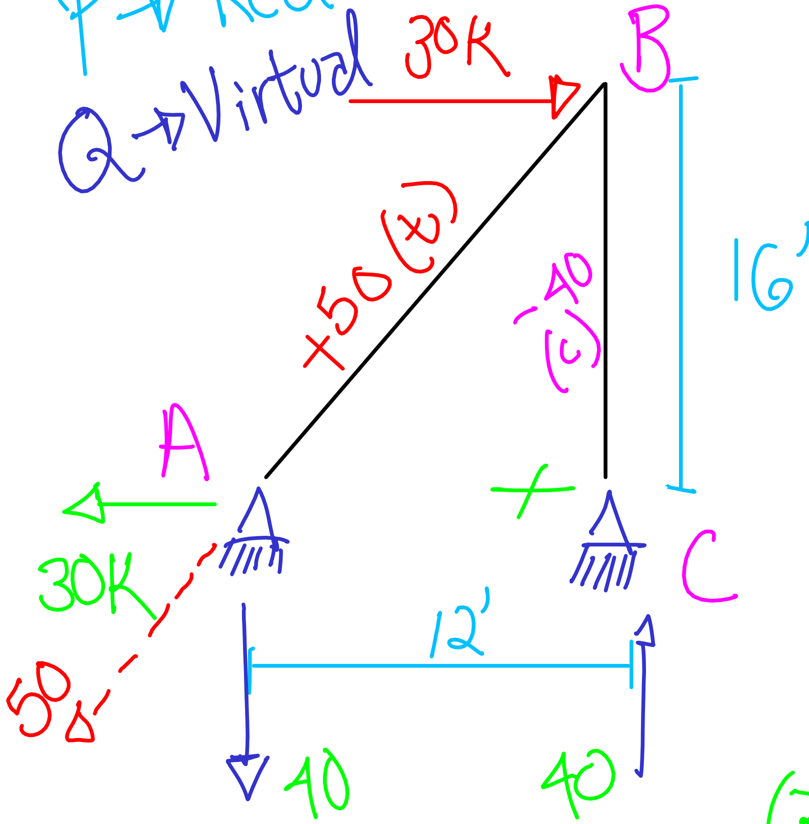
P → Real

Q → Virtual



Determinar el desplazamiento horizontal y vertical en B.

$E = 30{,}000 \text{ ksi}$

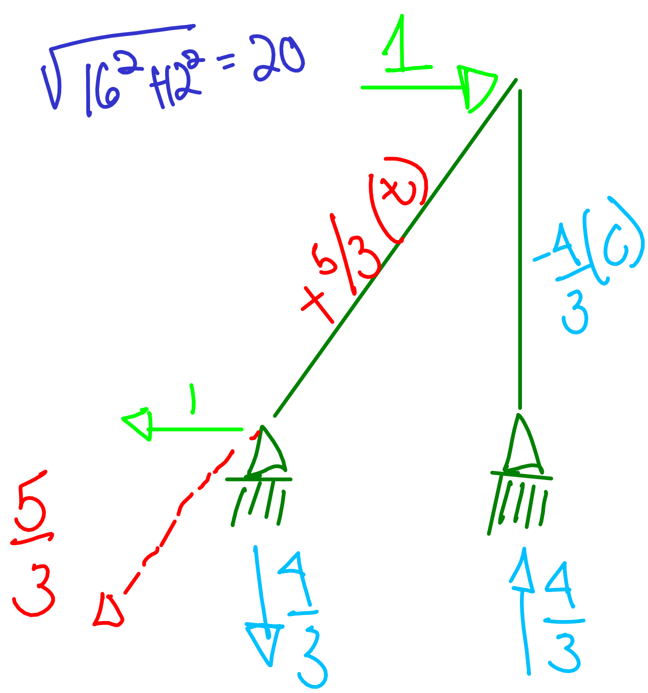
$A = 2 \text{ in}^2$

$$\sum Q\,\delta_p = \sum F_Q \frac{F_p L}{AE}$$

$$(30)(16) = x(12) \rightarrow x = 40$$

$$\sqrt{30^2 + 40^2} = 50$$

$$\sqrt{16^2 + 12^2} = 20$$

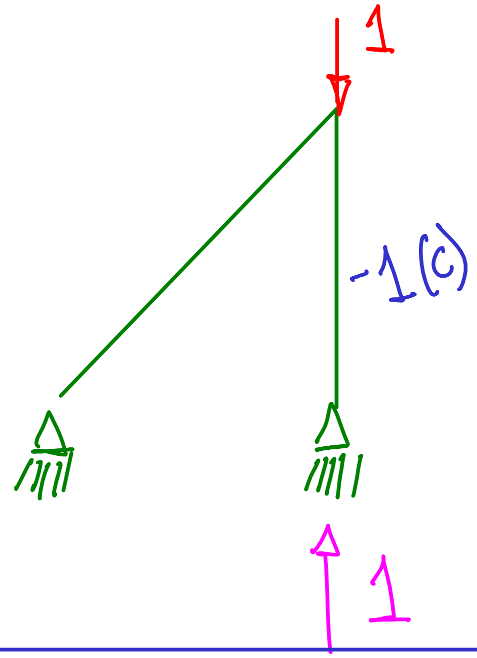


$$(1)(16) = x(12) \rightarrow x = \frac{4}{3}$$

$$\sqrt{\left(\frac{4}{3}\right)^2 + 1^2} = \frac{5}{3}$$

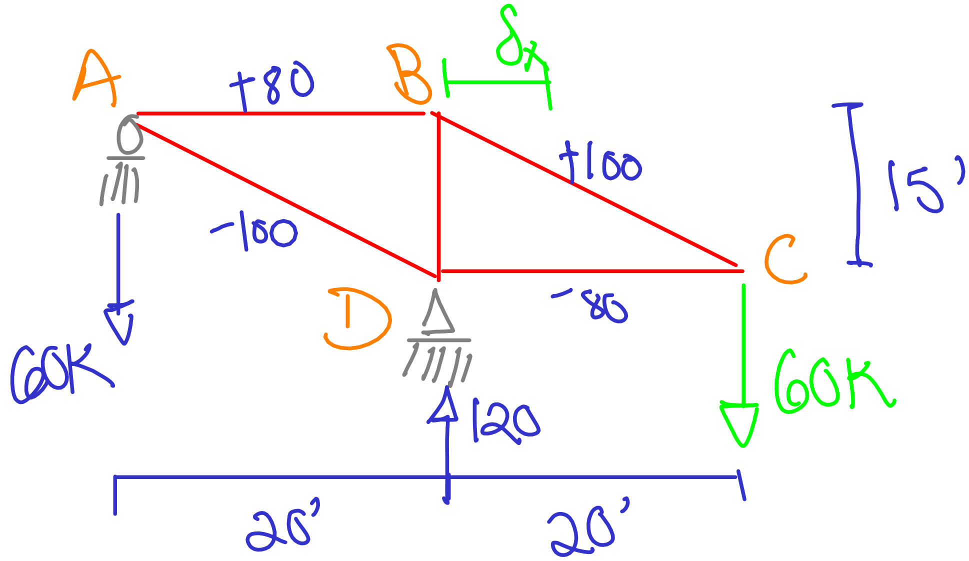
$$1\,\delta_p = \frac{5}{3}\left(\frac{50 \times 20 \times 12}{2 \times 30{,}000}\right) - \frac{4}{3}\left(\frac{-40 \times 16 \times 12}{2 \times 30{,}000}\right)$$

$$\delta_p = 0.5 \text{ in} \rightarrow$$



$$1\delta_p = -1\left(\frac{-40\times 16\times 12}{2\times 30{,}000}\right) + 0$$

$$\delta_p = 0.128 \text{ in} \downarrow$$

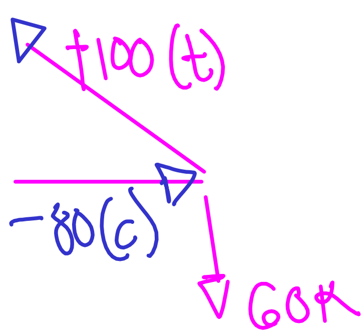


$E = 30{,}000$ Ksi

A_{AD} & $A_{BC} = 5 \text{ in}^2$

Demás áreas $= 4 \text{ in}^2$

Nodo C

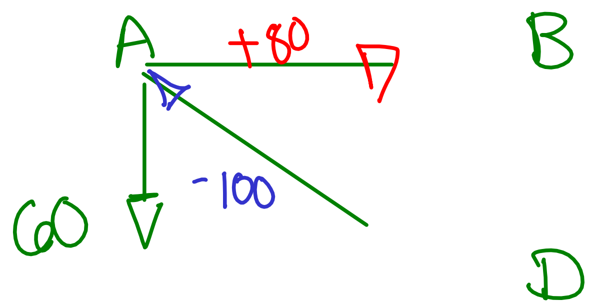


$$\Sigma F_y = -60 + F_{BC}\left(\frac{15}{25}\right) = 0 \rightarrow F_{BC} = 100\,T$$

$$\Sigma F_x = -100\left(\frac{20}{25}\right) + F_{DC} = 0$$

$$F_{DC} = -80 \rightarrow 80(C)$$

Nodo A



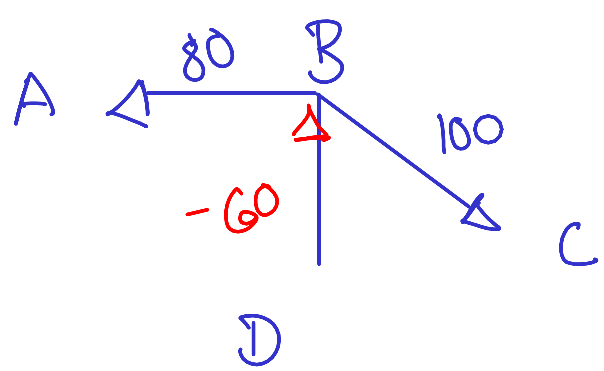
$$\Sigma F_y = -60 + F_{AD}\left(\frac{15}{25}\right) = 0$$

$$F_{AD} = 100\ (C)$$

$$\Sigma F_x = -100\left(\frac{20}{25}\right) + F_{AB} = 0$$

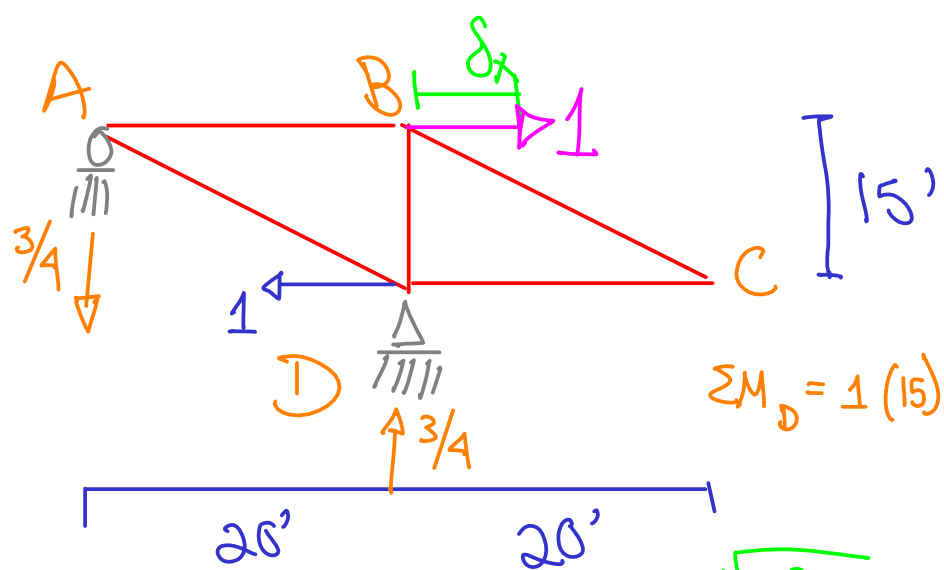
$$F_{AB} = 80\ T$$

Nodo B



$$\Sigma F_y = -100\left(\frac{15}{25}\right) + F_{BD} = 0$$

$$F_{BD} = 60\ (C)$$

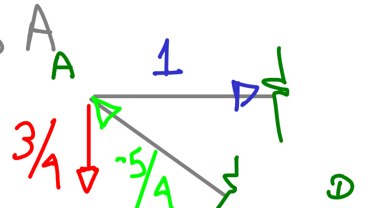


$$\Sigma M_D = 1(15) - x(20) = 0$$

$$x = \frac{15}{20} = \frac{3}{4}$$

$$\sqrt{15^2 + 20^2} = 25$$

Nodo A



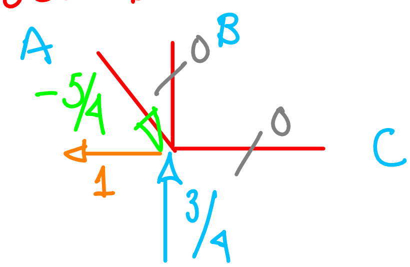
$$\Sigma F_y = \frac{-3}{4} + F_{AD}\left(\frac{15}{25}\right) = 0$$

$$F_{AD} = 5/4\ (C)$$

$$\Sigma F_x = -\frac{5}{4}\left(\frac{20}{25}\right) + F_{AB} = 0$$

$$\hookrightarrow F_{AB} = \underline{1}\ (T)$$

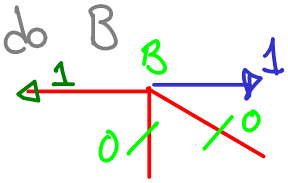
Nodo D



$$\frac{5}{4}\left(\frac{20}{25}\right) = 1$$

$$\frac{5}{4}\left(\frac{15}{25}\right) = \frac{3}{4}$$

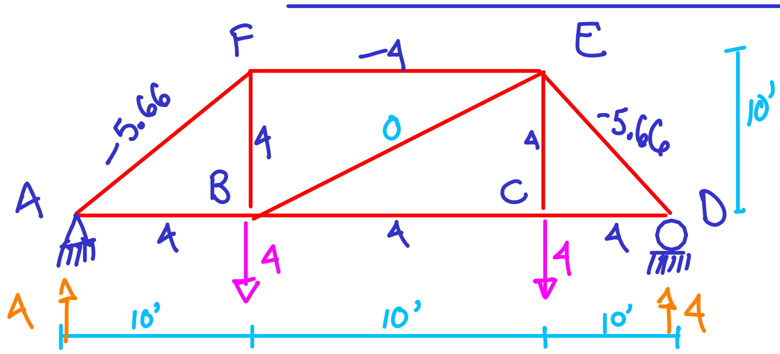
Nodo B



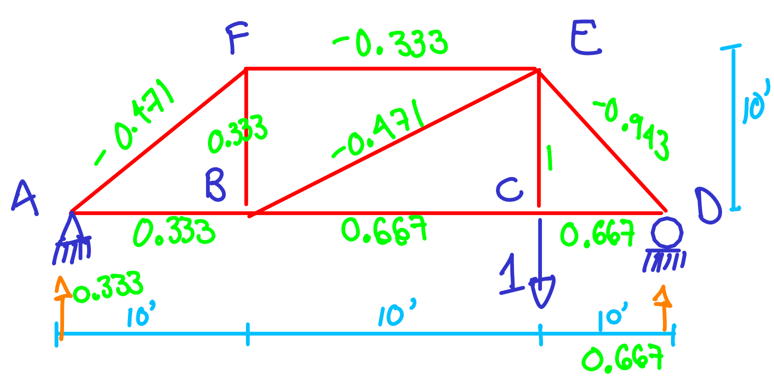
Miembro	F_Q	F_p	L	A	F_pF_QL/A
AB	+1	+80	20	4	+400
BC	0	+100	25	5	0
AD	-5/4	-100	25	5	+625
BD	0	-60	15	4	0
DC	0	-80	20	4	0
					1025

$$\Sigma F_Q \frac{F_p L}{AE} = \Sigma Q \delta_p$$

$$\frac{1025}{30{,}000}(12) = 1\,\delta_p \qquad \underline{\underline{\delta_p = 0.41 \text{ in}}}$$



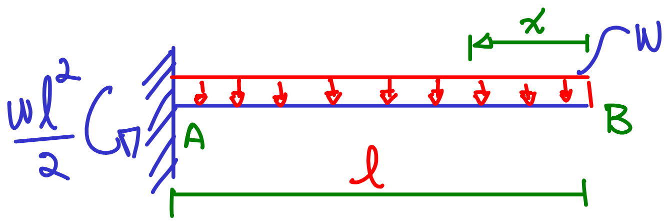
Determine el desplazamiento vertical
de la junta C de la armadura de acero.
El área de la sección transversal de cada
miembro es de A = 0.5 in², y E = 29,000 ksi.



Miembro	F_Q	F_p	L	A	F_pF_QL/A
AB	0.333	4	10	0.5	26.64
BC	0.667	4	10	0.5	53.36
CD	0.667	4	10	0.5	53.36
DE	-0.943	-5.66	14.14	0.5	160.94
FE	-0.333	-4	10	0.5	26.64
EB	-0.471	0	14.14	0.5	0
BF	0.333	4	10	0.5	26.64
AF	-0.471	-5.66	14.14	0.5	75.39
CE	1	4	10	0.5	80
					493

$$\Sigma F_Q \frac{F_pL}{AE} = \Sigma Q\,\delta_p$$

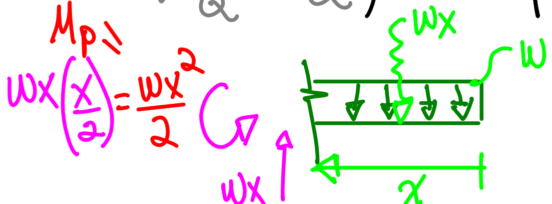
$$\frac{493}{29{,}000}(12) = 1\,\delta_p \qquad \underline{\underline{\delta_p = 0.204 \text{ in}}}$$



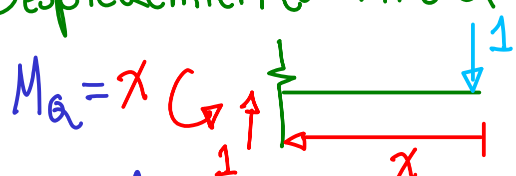
Utilizando el trabajo virtual, calcule la deflexión y la rotación en B para la viga con carga uniformemente repartida. (Viga en voladizo, viga en cantilever).

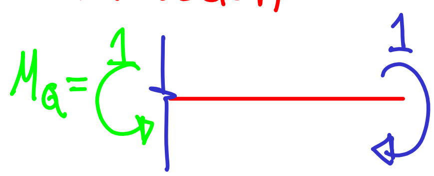
Viga real

$$W_Q = U_Q \,;\ \Sigma Q\delta_p = \int M_Q \frac{M_p\,dx}{EI}$$



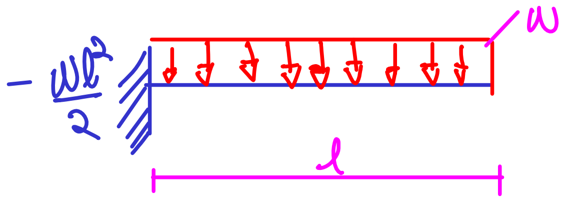
Desplazamiento Virtual. | Rotacion Virtual





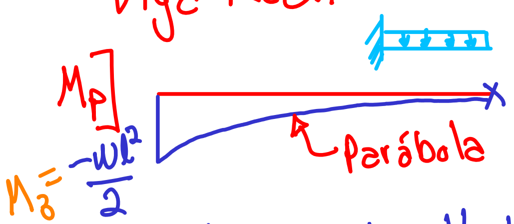
$$EI\,1\,\delta_p = \int_0^l (x)\left(\frac{wx^2}{2}\right)dx = \frac{wx^4}{8}\Big|_0^l = \frac{wl^4}{8} \rightarrow \delta_B = \frac{wl^4}{8EI}\downarrow$$

$$EI\,\theta_p = \int_0^\ell (1)\left(\frac{wx^2}{2}\right)dx = \frac{wx^3}{6}\Big|_0^\ell = \frac{w\ell^3}{6} \to \theta_B = \frac{w\ell^3}{6EI}$$

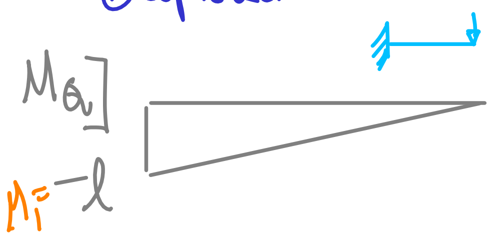


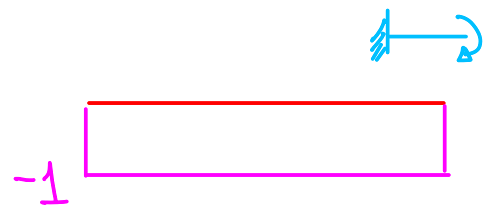
Lo mismo pero con las áreas de los diagramas y las tablas de productos de integrales.

Viga Real.



Desplazamiento Virtual. Rotación Virtual.





Del Libro $\to \int_0^\ell M_Q M_p\,dx$

$$= \frac{1}{4} M_1 M_3 L = \frac{1}{4}(-\ell)\left(\frac{-w\ell^2}{2}\right)\ell = \frac{w\ell^4}{8}$$

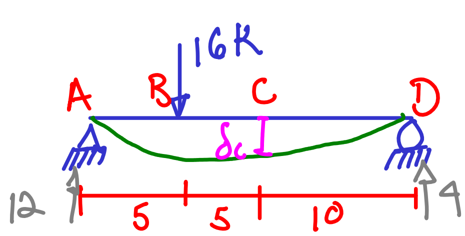
$$1\,\delta_p = \int_0^L \frac{M_Q M_p}{EI}dx \to 1\,EI\,\delta_p = \frac{w\ell^4}{8}$$

$$\delta_B = \frac{w\ell^4}{8EI}$$

Del Libro $\to \int M_Q M_p\,dx = \frac{1}{3} M_1 M_3 L$

$$\frac{1}{3}(-1)\left(-\frac{w\ell^2}{2}\right)\ell = \frac{w\ell^3}{6}$$

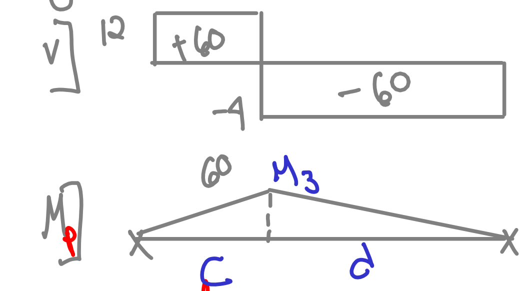
$$EI\,1\,\theta_p = \int M_p M_Q\,dx \to \theta_B = \frac{w\ell^3}{6EI}$$



$E = 29{,}000$ $I = 240\ in^4$

¿δ_c?

Viga Real



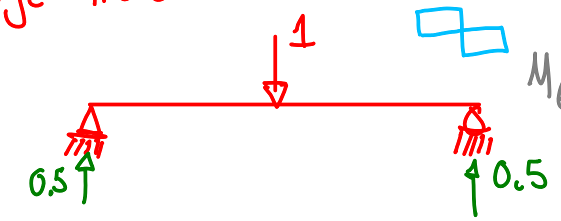
$\Sigma M_A = 16(5) - D_y(20) = 0$

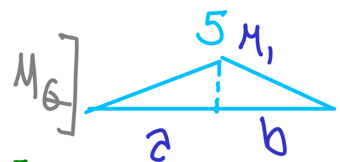
$D_y = 4k$

$\Sigma F_y = 16 - 4 - A_y = 0$

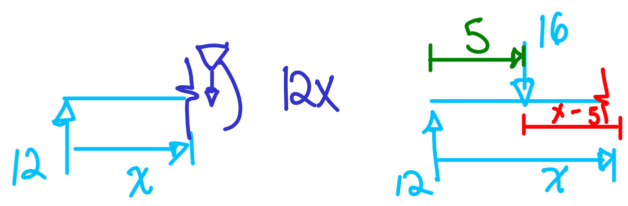
$A_y = 12k$

Viga Virtual





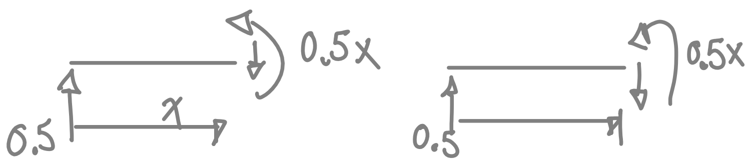
Segmento	Origen	Rango	M_Q	M_P
AB	A	0-5	$0.5x$	$12x$
BC	A	5-10	$0.5x$	$-4x+80$
DC	D	0-10	$0.5x$	$4x$

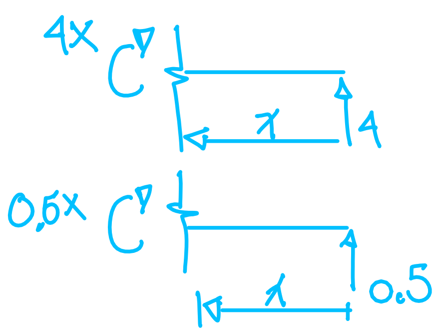


$M = 12x - 16(x-5)$

$M = 12x - 16x + 80$

$M = -4x + 80$





$$EI\, 1\,\text{Kip}\, \delta_{P_C} = \int_0^5 (12x)(0.5x)\,dx + \int_5^{10} (-4x+80)(0.5x)\,dx + \int_0^{10} (4x)(0.5x)\,dx$$

$$\frac{6x^3}{3}\Big|_0^5 + \int_5^{10} -2x^2 + 40x\,dx + \frac{2x^3}{3}\Big|_0^{10}$$

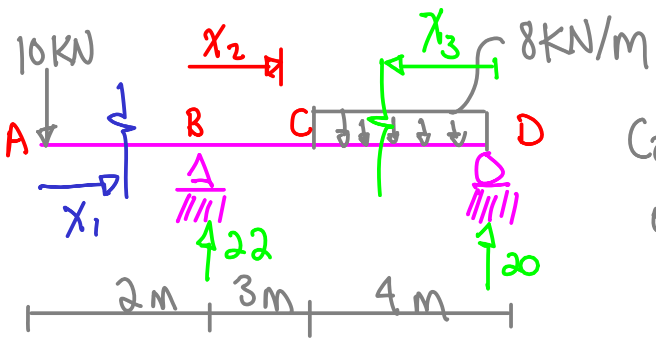
$$250 + \frac{-2x^3}{3} + \frac{40x^2}{2}\Big|_5^{10} + 666.67 = 250 + 1333.33 + 666.67 - 416.66 = 1833.33$$

$$EI\,1\,\delta_{p_c} = 1833.33$$

$$\delta_{p_c} = \frac{1833.33\,(12^3)}{29{,}000\,(240)} = 0.45\text{ in}$$

Tablas de Integrales

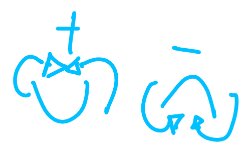
$$\left(\frac{1}{3} - \frac{(a-c)^2}{6ad}\right) M_1 M_3 L = \left(\frac{1}{3} - \frac{(10-5)^2}{6(10)(15)}\right)(5)(60)(20)\left(\frac{12^3}{29{,}000 \times 240}\right) = 0.45\text{ in}$$

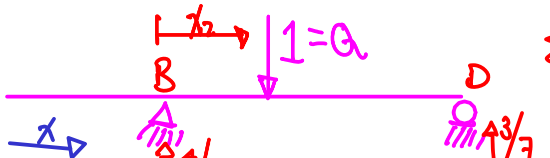


Calcule la deflexion en C ; EI = Constante.

$$\Sigma M_B = 10(2) - 8(4)(5) + D_y(7) = 0 \rightarrow D_y = 20$$

$$\Sigma F_y = -10 - 8(4) + 20 + B_y = 0 \rightarrow B_y = 22$$

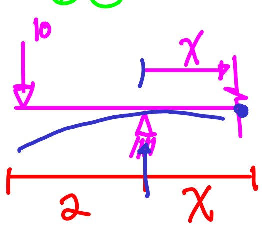


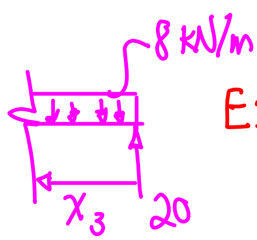


$$\Sigma M_B = 1(3) - D_y(7) = 0$$

$$D_y = 3/7$$

Segmento	Origen	Limites	M_p	M_Q	
AB	A	0-2	$-10x$	0	
BC	B	0-3	$-10(x+2)+22x$	$(4/7)x$	$-10x-20+22x = 12x-20$
DC	D	0-4	$20x - 8x\left(\frac{x}{2}\right)$	$(3/7)x$	$20x - 4x^2$





$$EI\,1\,\delta_{p_c} = \cancel{\int_0^2 -10x(0)\,dx} + \int_0^3 (12x-20)\left(\tfrac{4}{7}x\right)dx + \int_0^4 (20x-4x^2)\left(\tfrac{3}{7}x\right)dx$$

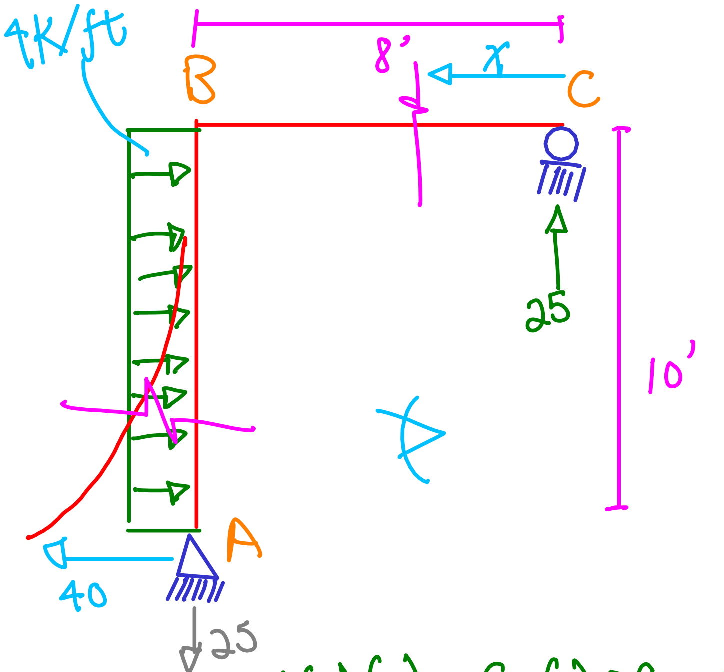
$$\frac{48}{7}x^2 - \frac{80}{7}x$$

$$\frac{60}{7}x^2 - \frac{12}{7}x^3$$

$$EI\delta_{p_c} = \left(\frac{48x^3}{21} - \frac{80x^2}{14}\right)\Bigg|_0^3 + \left(\frac{60x^3}{21} - \frac{12x^4}{28}\right)\Bigg|_0^4$$

$$= 10.286 + 73.14 = 83.43$$

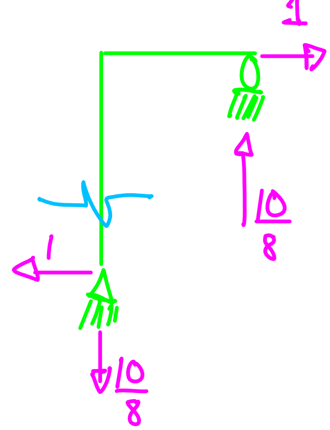
$$\delta_{p_c} = 83.43/EI \downarrow$$



$\delta_{Horiz.\,C}$

$E = 29{,}000$ ksi

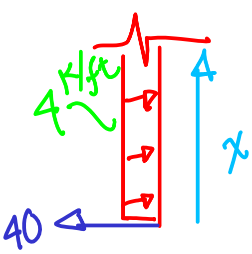
$I = 600\ in^4$



$$\Sigma M_A = 4(10)(5) - C_y(8) = 0 \rightarrow C_y = 25\,K$$

$$\Sigma F_x = 4(10) - A_x = 0 \rightarrow A_x = 40$$

Segmento	Origen	Limites	M_P	M_Q
AB	A	0-10	$\underbrace{40x - 4x\left(\frac{x}{2}\right)}_{\alpha}$	$1x$
CB	C	0-8	$25x$	$\frac{10}{8}x$



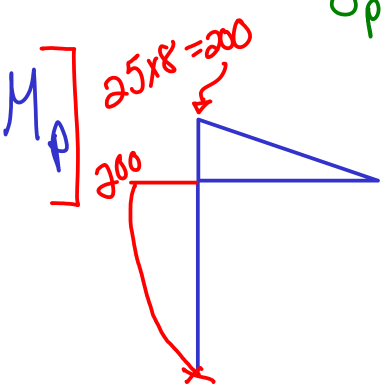
$$\alpha = 40x - 2x^2$$

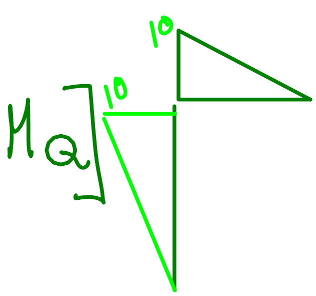
$$EI\,1\,\delta_{Ph_C} = \int_0^{10} \underbrace{(40x-2x^2)(x)}_{\beta}dx + \int_0^{8} \underbrace{(25x)(1.25x)}_{\gamma}dx$$

$$\beta = 40x^2 - 2x^3 \qquad \gamma = 31.25x^2$$

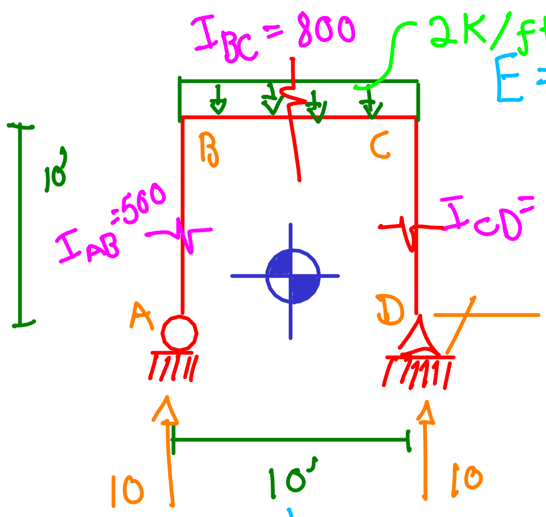
$$EI\,1\,\delta_{Ph_C} = \left(\frac{40x^3}{3} - \frac{2x^4}{4}\right)\Big|_0^{10} + \frac{31.25x^3}{3}\Big|_0^{8} = 8{,}333.33 + 5{,}333.33$$

$$\delta_{Ph_C} = \frac{13{,}666.67}{EI} = \frac{13666.67\,(12^3)}{29{,}000 \times 600} = 1.357\text{ in} \rightarrow$$

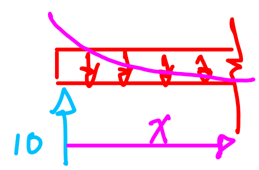




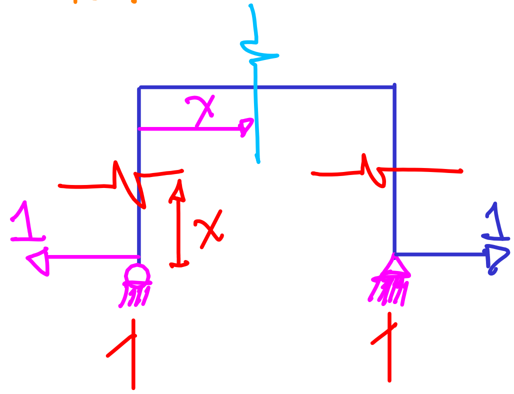
Obtener mediante áreas.

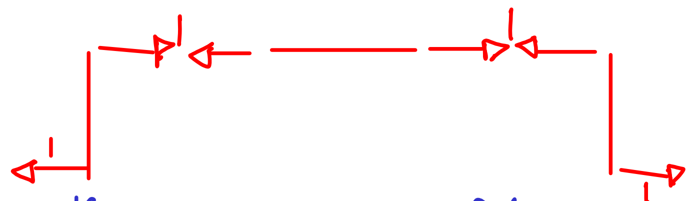


$\delta_{Horiz. A} = ¿?$



Segmento	Origen	Límites	M_p	M_Q
~~AB~~	A	0-10	0	1x
BC	B	0-10	$10x - 2x\left(\frac{x}{2}\right)$	10
~~CD~~	D	0-10	0	1x

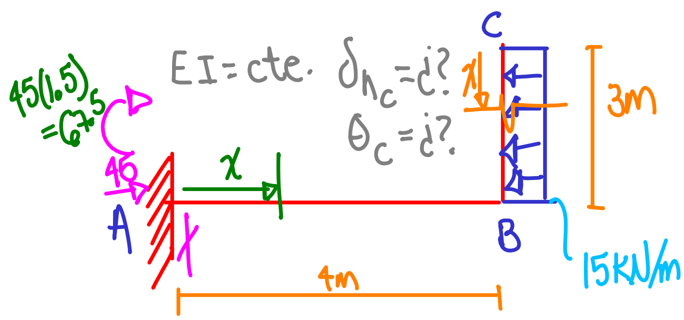


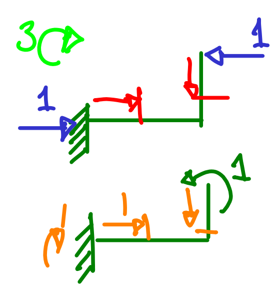


$$EI\,\delta_{h_A} = \int_0^{10} (10x - x^2)(10)\,dx = \int_0^{10} 100x - 10x^2\,dx$$

$$= \left(\frac{100x^2}{2} - \frac{10x^3}{3}\right)\Big|_0^{10} = 1666.67$$

$$\delta_{h_A} = \frac{1666.67\,(12^3)}{(29{,}000)(800)} = 0.124\ in \leftarrow$$





Segmento	Origen	Límite	M_p	M_Q^{δ}	M_Q^{θ}
AB	A	0-4	67.5	3	1
CB	C	0-3	$15x\left(\frac{x}{2}\right)$	x	1

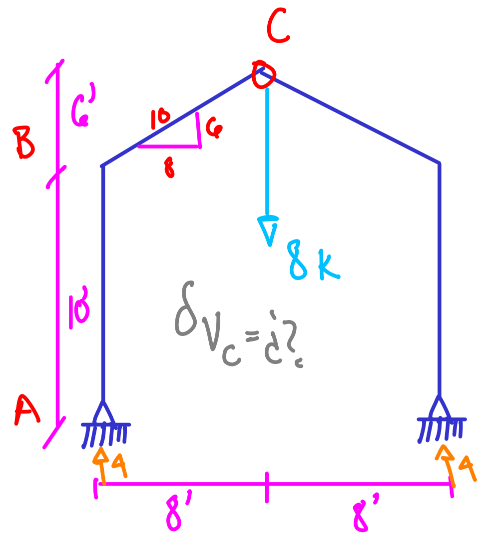
$$EI\delta_{h_c} = \int_0^4 (67.5)(3)\,dx + \int_0^3 7.5x^2 \cdot x\,dx = 202.5x\Big|_0^4 + \frac{7.5x^4}{4}\Big|_0^3$$

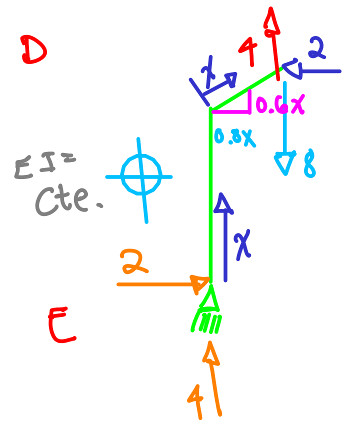
$$\delta_c = 961.875/EI \leftarrow 810 + 151.875 = 961.875$$

$$EI\theta_C = \int_0^4 67.5(1)\,dx + \int_0^3 7.5x^2(1)\,dx$$

$$= 67.5x\Big|_0^4 + \frac{7.5x^3}{3}\Big|_0^3 = 270 + 67.5 = \underline{337.5}$$

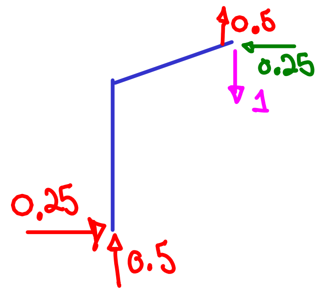
$\theta = 337.5/EI$ ↺





$\Sigma M_C = 4(8) - A_x(16) = 0$

$A_x = 2$



$\Sigma M_C = 0.5(8) - A_x(16) = 0$

$A_x = 0.25$

Segmento	Origen	Límites	M_P	M_Q
AB	A	0-10	$2x$	$0.25x$
BC	B	0-10	$20-2x$	$2.5-0.25x$

$20 - 0.8x(4) + 2(0.6x)$

$20 - 3.2x + 1.2x$

$20 - 2x$

$2.5 - 0.8x(0.5) + 0.25(0.6x)$

$2.5 - 0.4x + 0.15x$

$2.5 - 0.25x$

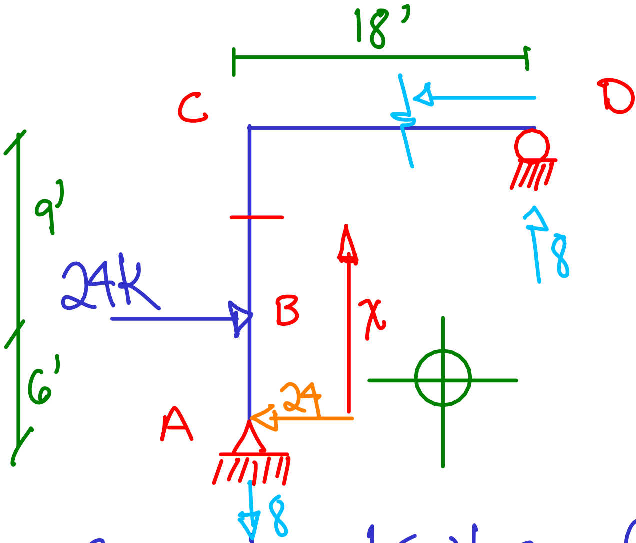
$$EI\,\delta_{V_C} = 2\left(\int_0^{10} 2x(0.25x)\,dx + \int_0^{10} (20-2x)(2.5-0.25x)\,dx\right)$$

$$2\left(\frac{0.5x^3}{3}\Big|_0^{10} + \int_0^{10} 50 - 5x - 5x + 0.5x^2\,dx\right)$$

$$2\left(166.67 + \left(50x - \frac{10x^2}{2} + \frac{0.5x^3}{3}\right)\Big|_0^{10}\right)$$

$$= 2\left(166.67 + 500 - 500 + \frac{500}{3}\right) = 666.67$$

$$\delta_{V_C} = -\frac{666.67}{EI}$$

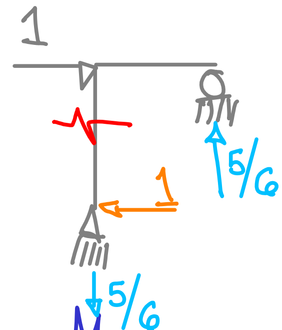


$I = 600\ in^4$

$A = 13\ in^2$

$E = 29{,}000$

$\delta_{h_C} = ¿?$



Segmento	Limites	Origen	M_P	M_Q
AB	0-6	A	$24x$	$1x$
BC	6-15	A	$24x-24(x-6)$	$1x$
DC	0-18	D	$8x$	$\frac{5}{6}x$

$$\delta_{h_C} = \int_0^6 \frac{(24x)(x)\,dx}{EI} + \int_6^{15} \frac{(144)(x)\,dx}{EI} + \int_0^{18} \frac{(8x)(\frac{5}{6}x)\,dx}{EI}$$

$$+ \frac{(8)(\frac{5}{6})(15 \times 12)}{AE}$$

Flexion = 2.8 in

Axial = 0.0032 in