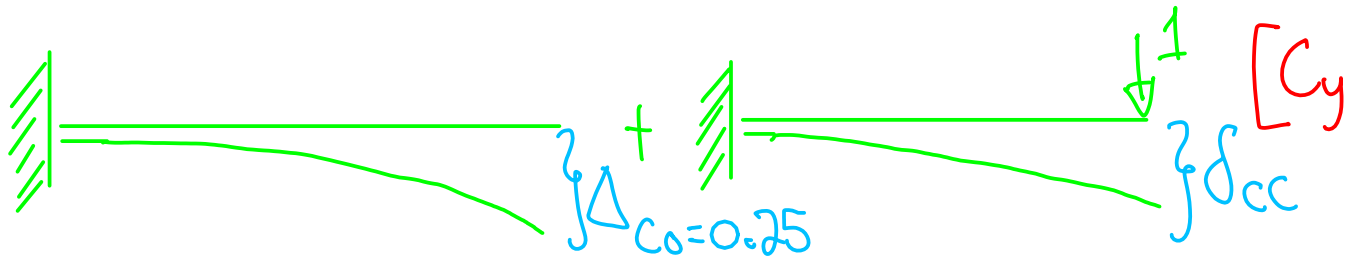
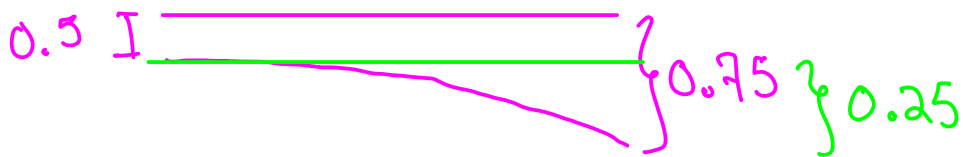


Asumiendo que no actúa ninguna carga, calcule las reacciones y dibuje los diagramas de fuerza cortante y momento flexionante para la viga de la figura si el apoyo A se asienta 0.5 in y el apoyo C se asienta 0.75 in. Se da $E = 29,000$ ksi; $I = 150$ in⁴.

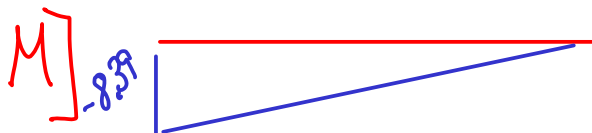
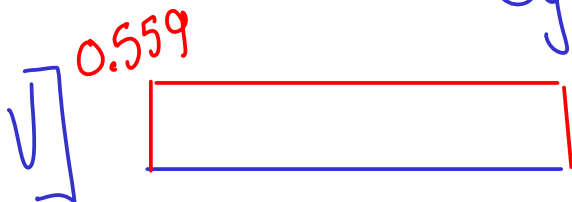


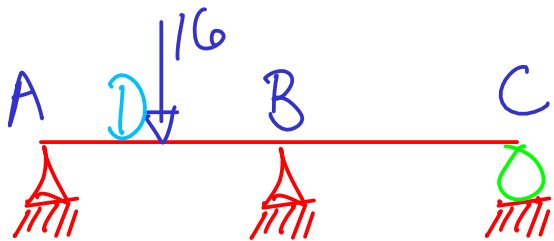
$$\delta_{CC} = \frac{Pl^3}{3EI} = \frac{1(15^3)(1728)}{3(29000)(150)} = 0.447 \text{ in}$$

$$\Delta_{C0} + \delta_{CC} C_y = 0$$

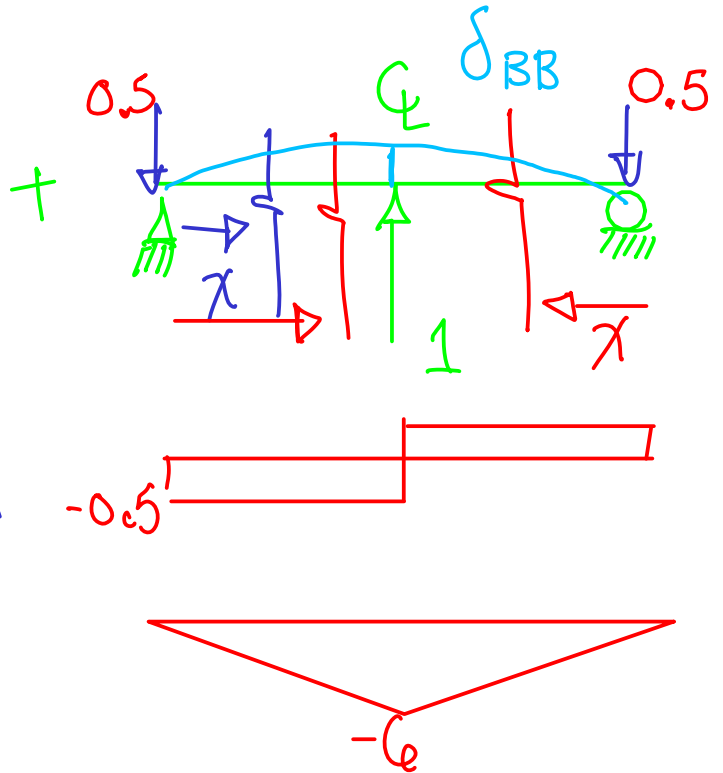
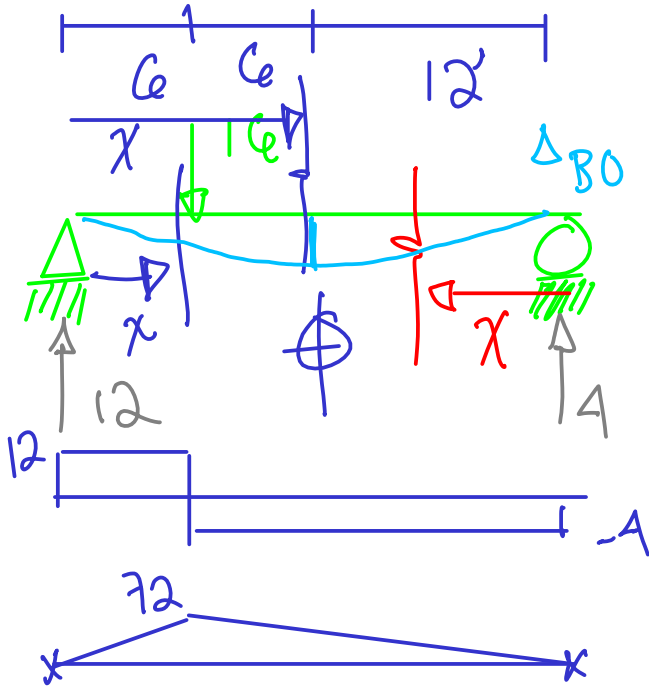
$$0.25 + 0.447 C_y = 0$$

$$C_y = 0.559 \text{ Kips } \downarrow$$





Calcule las reacciones para la viga mostrada. Asuma que los apoyos no se mueven; El es constante. Repita los cálculos si es que el apoyo C se mueve hacia arriba una distancia de $288/EI$ cuando se aplica la carga.



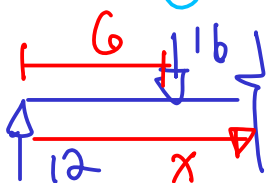
$$\sum M_A = 16(6) - C_y(24) = 0$$

$$C_y = 4$$

$$\sum F_y = 16 - 4 - A_y = 0 \therefore A_y = 12$$

Trabajo Virtual

Segmento	Origen	Limites	M_p	M_a
AD	A	0-6	$12x$	$-0.5x$
DB	A	6-12	$-4x+96$	$-0.5x$
BC	C	0-12	$4x$	$-0.5x$



$$12x - 16(x-6) = 12x - 16x + 96 = -4x + 96$$

$$\int_0^6 12 \cdot x \cdot (-0.5 \cdot x) dx = -432$$

$$\int_6^{12} (-4 \cdot x + 96) \cdot (-0.5 \cdot x) dx = -1584$$

$$\int_0^{12} 4 \cdot x \cdot (-0.5 \cdot x) dx = -1152$$

$$\Delta_{B0} = \frac{-432 - 1584 - 1152}{EI}$$

$$\Delta_{B0} = -\frac{3168}{EI} = \frac{3168}{EI} \downarrow$$

$$\int_0^6 (-0.5 \cdot x) \cdot (-0.5 \cdot x) dx = 18$$

$$\int_6^{12} (-0.5 \cdot x) \cdot (-0.5 \cdot x) dx = 126$$

$$\int_0^{12} (-0.5 \cdot x) \cdot (-0.5 \cdot x) dx = 144$$

$$\delta_{BB} = \frac{18 + 126 + 144}{EI}$$

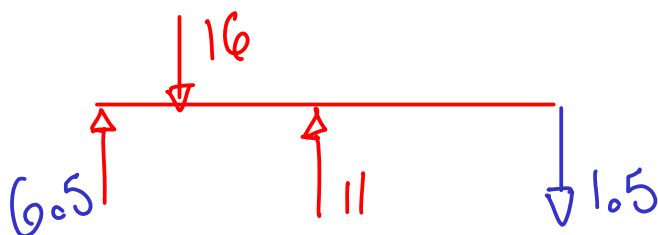
$$= \frac{288}{EI} \uparrow$$

Ec. de compatibilidad

$$\Delta_{B0} + \delta_{BB} B_y = 0$$

$$-\frac{3168}{EI} + \frac{288}{EI} B_y = 0$$

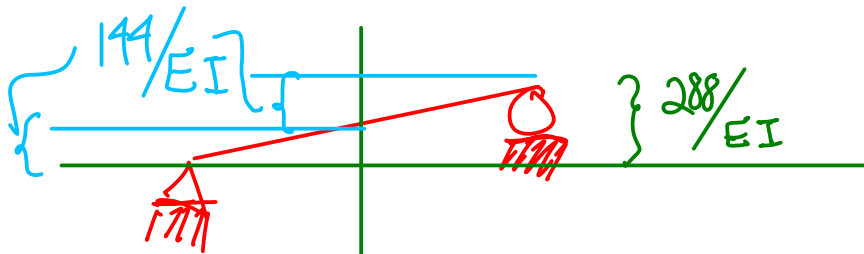
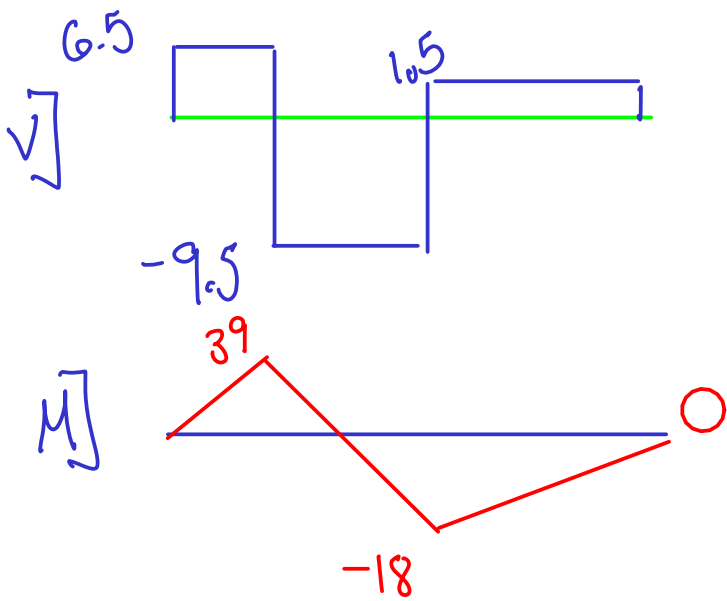
$$B_y = 11 \text{ K} \uparrow$$



$$\sum M_A = 16(6) - 11(12) + C_y(24) = 0$$

$$C_y = 1.5$$

$$\sum F_y = A_y + 11 - 1.5 - 16 = 0 \mid A_y = 6.5$$

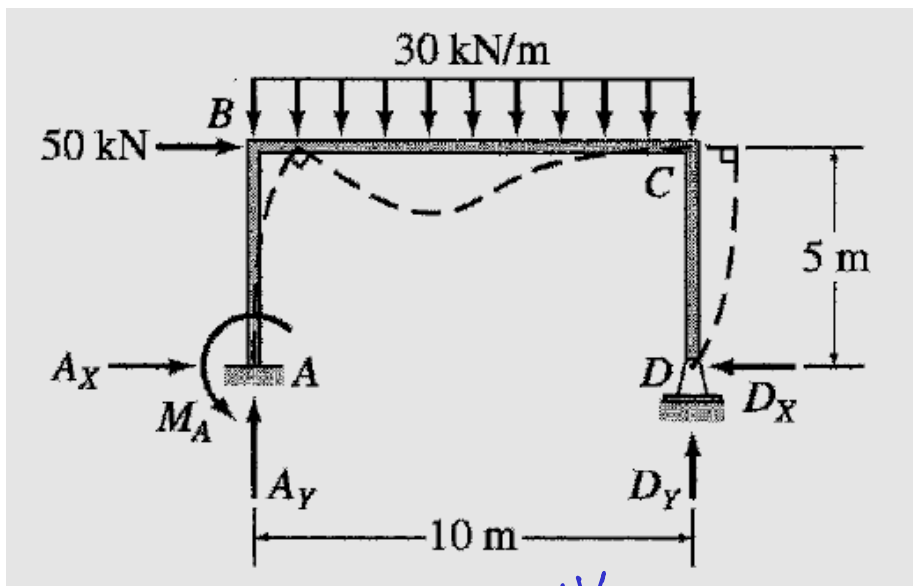


Ec. de compatibilidad

$$\Delta_{B0} + \delta_{BB} B_y = -\frac{144}{EI}$$

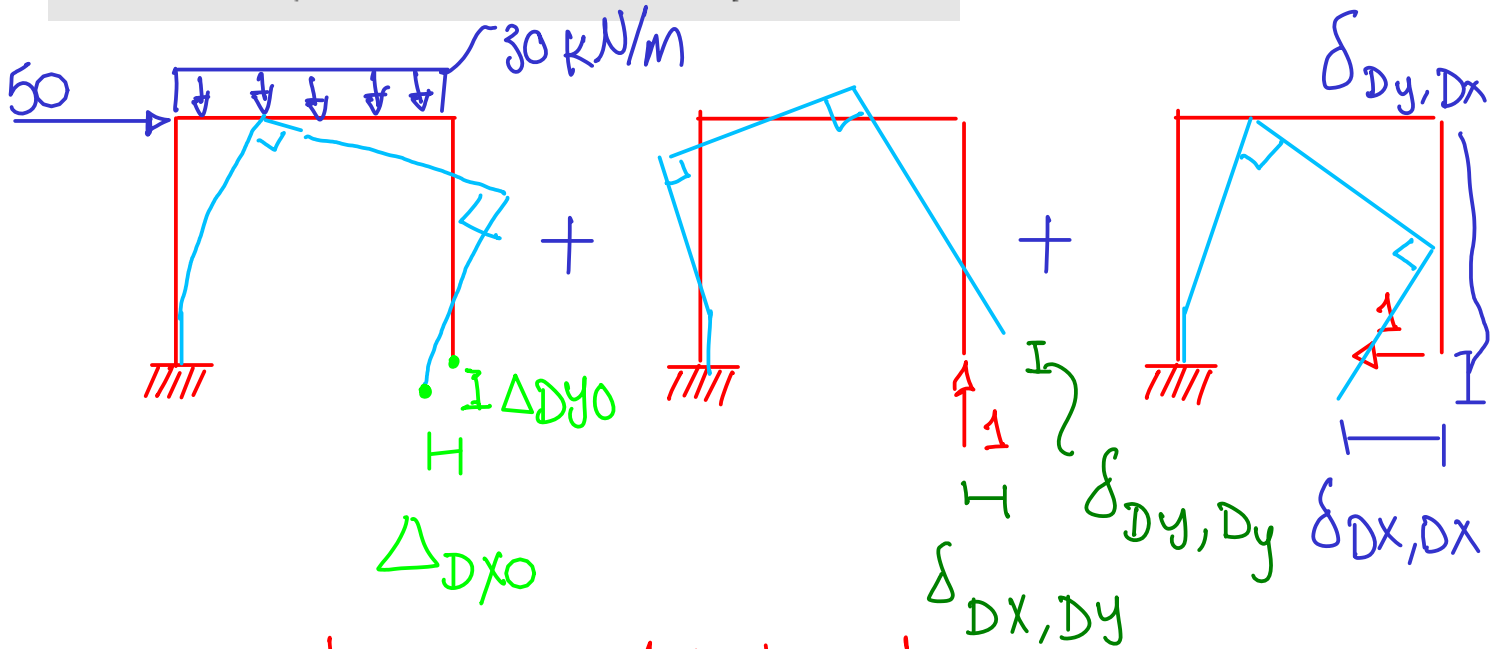
$$-\frac{3168}{EI} + \frac{288}{EI} B_y = -\frac{144}{EI} \quad B_y = \underline{10.5K\uparrow}$$

$$\left(\frac{1}{3} - \frac{(a-c)^2}{6ad} \right) M_1 M_3 L = \left(\frac{1}{3} - \frac{(12-6)^2}{6(12)(18)} \right) (6)(72)(2A) = \frac{3168}{EI}$$



Reacciones

V
&
M



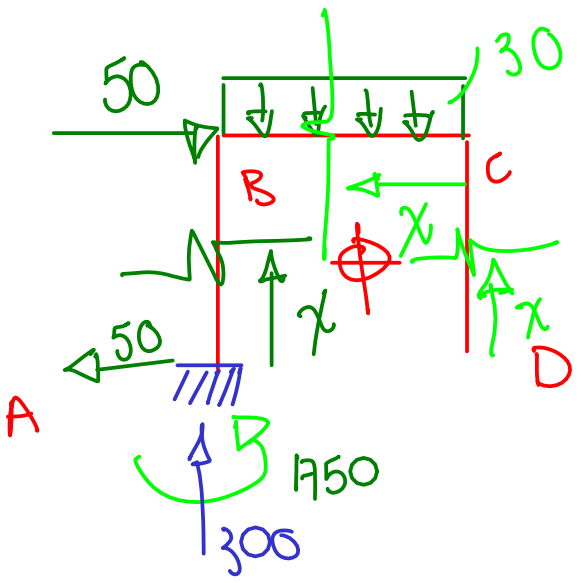
Ec. de compatibilidad

$$\textcircled{1} \quad \Delta_{Dx0} + \delta_{DxDx} D_x + \delta_{DxDy} D_y = 0$$

$$\textcircled{2} \quad \Delta_{Dy0} + \delta_{DyDx} D_x + \delta_{DyDy} D_y = 0$$

Segmento Origen Limites M_p M_{Q_y} M_{Q_x}

AB	A	0-5	$-1750 + 50x$	10	$-x$
BC	C	0-10	$-15x^2$	x	-5
CD	D	0-5	0	0	$-x$

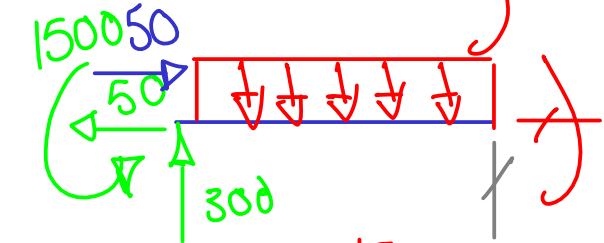
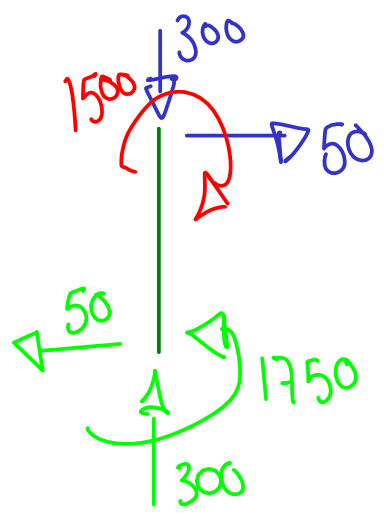
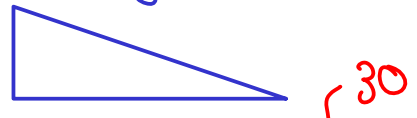


$$\sum M_A = M_A - 50(5) - (30)(10)(5) = 0$$

$$M_A = 1750 \text{ kNm } \curvearrowleft$$

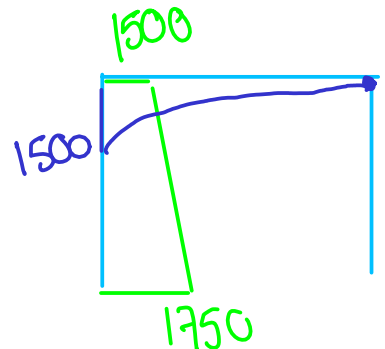
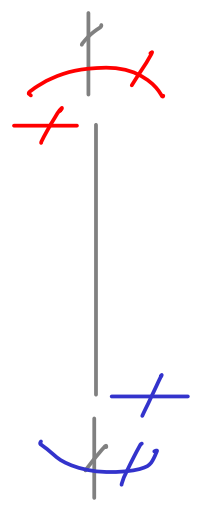
$$\sum F_x = 50 - A_x = 0 \therefore A_x = 50 \curvearrowleft$$

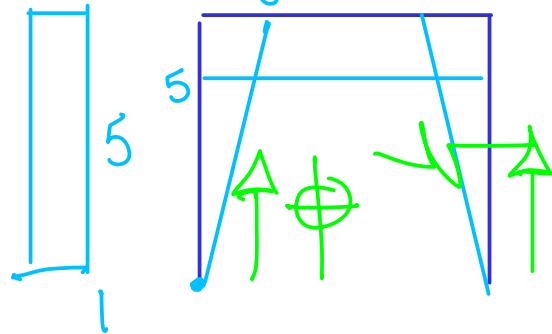
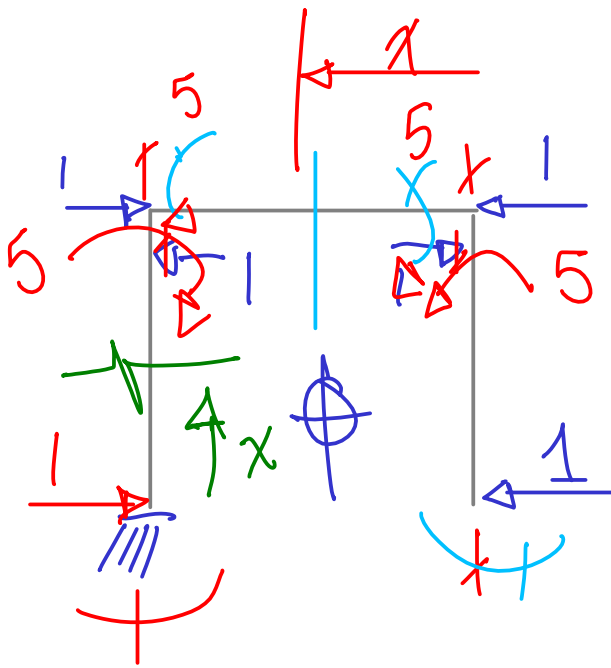
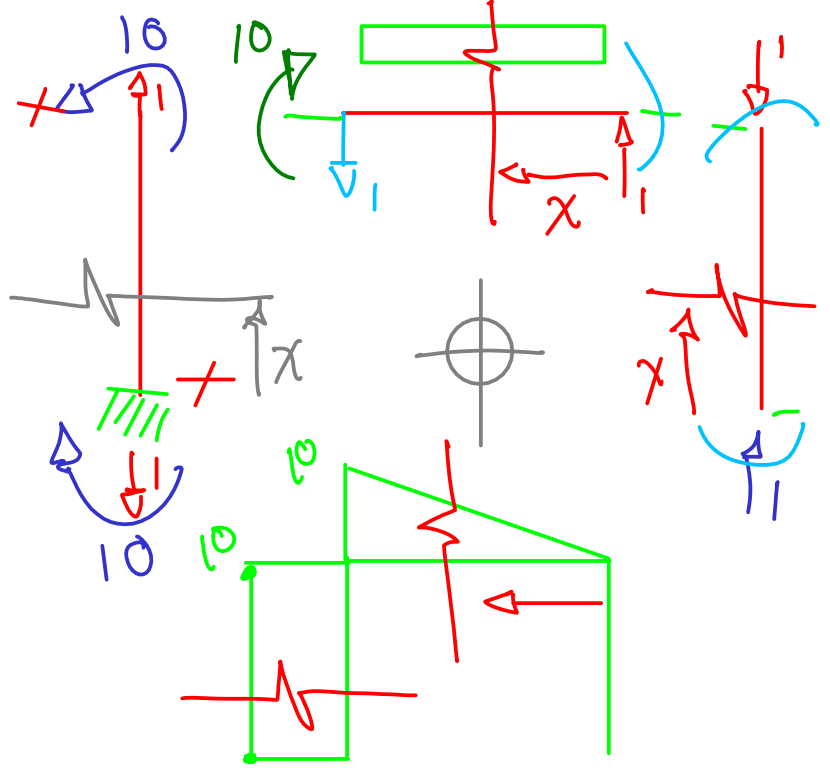
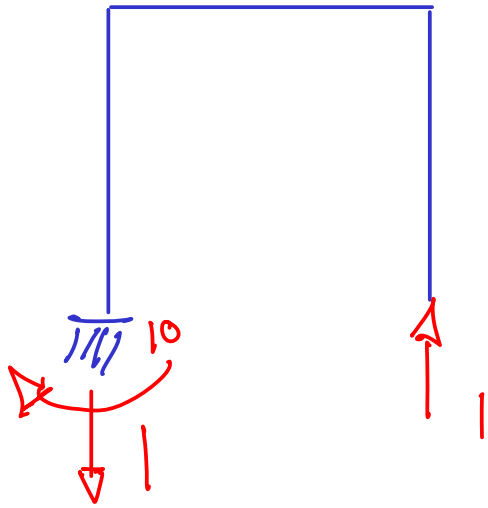
$$\sum F_y = -30(10) + A_y = 0 \therefore A_y = 300 \uparrow$$



$$250 - 1750 = 1500$$

$$1500 - 30(10)(5) = 0$$





$$EI \Delta_{x0} = \int_0^5 \underline{(-1750 + 50x)}(-x) dx + \int_0^{10} \underline{(-15x^2)}(-5) dx$$

$$+ \int_0^5 \underline{(0)}(x) dx = 44791.67$$

$$A = \int_0^5 (-1750 + 50x) \cdot (-x) dx = 19791.6667$$

$$B = \int_0^{10} (-15x^2) \cdot (-5) dx = 25000$$

$$C = \int_0^5 (0) \cdot (x) dx = 0$$

$$A + B + C = 44791.6667$$

$$EI \Delta_{y0} = \int_0^5 (-1750 + 50x)(10) dx$$

$$+ \int_0^{10} (-15x^2)(x) dx + \int_0^5 (0)(0) dx = -118,750$$

$$EI \delta_{Dx Dx} = \int_0^5 (-x)(-x) dx + \int_0^{10} (-5)(-5) dx$$

$$+ \int_0^5 (-x)(-x) dx = 333.33$$

$$A := \int_0^5 (-1750 + 50 \cdot x) \cdot (10) dx = -81250$$

$$B := \int_0^{10} (-15 \cdot x^2) \cdot (x) dx = -37500$$

$$C := \int_0^5 (0) \cdot (0) dx = 0$$

$$A+B+C = -1.1875 \cdot 10^5$$

$$A := \int_0^5 (-x) \cdot (-x) dx = 41.6667$$

$$B := \int_0^{10} (-5) \cdot (-5) dx = 1$$

$$C := \int_0^5 (-x) \cdot (-x) dx = 41.6667$$

$$A+B+C = 333.3333$$

$$EI \delta_{Dy Dy} = \int_0^5 (10)(10) dx$$

$$+ \int_0^{10} (x)(x) dx + \int_0^5 (0)(0) dx$$

$$= 833.33$$

$$A := \int_0^5 (10) \cdot (10) dx = 500$$

$$B := \int_0^{10} (x) \cdot (x) dx = 333.3333$$

$$C := \int_0^5 (0) \cdot (0) dx = 0$$

$$A+B+C = 833.3333$$

$$\delta_{Dx Dy} = \delta_{Dy Dx}$$

$$= \int_0^5 (-x)(10) dx + \int_0^{10} (-5)(x) dx + \int_0^5 (-x)(0) dx$$

$$= -375$$

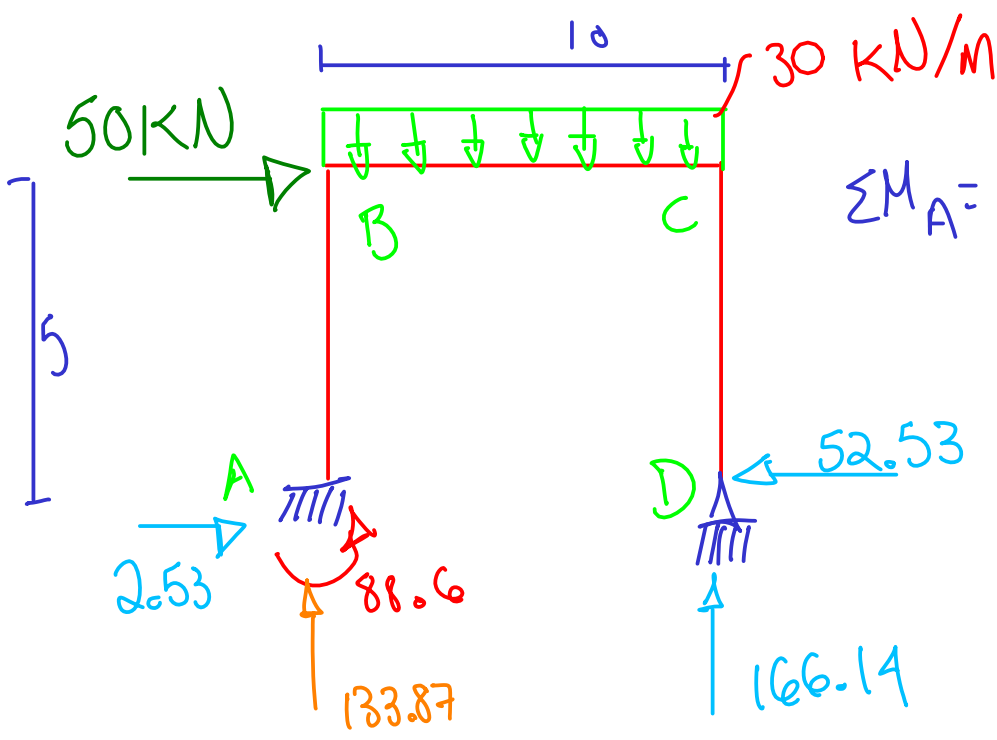
$$44791.67 + 333.33 D_x - 375 D_y = 0 \quad (1)$$

$$-118750 - 375 D_x + 833.33 D_y = 0 \quad (2)$$

$$\begin{bmatrix} 333.33 & -375 \\ -375 & 833.33 \end{bmatrix}^{-1} \begin{bmatrix} -44791.67 \\ 118750 \end{bmatrix} = \begin{bmatrix} 52.5342 \\ 166.1411 \end{bmatrix}$$

$$D_x = 52.53 \leftarrow$$

$$D_y = 166.14 \uparrow$$



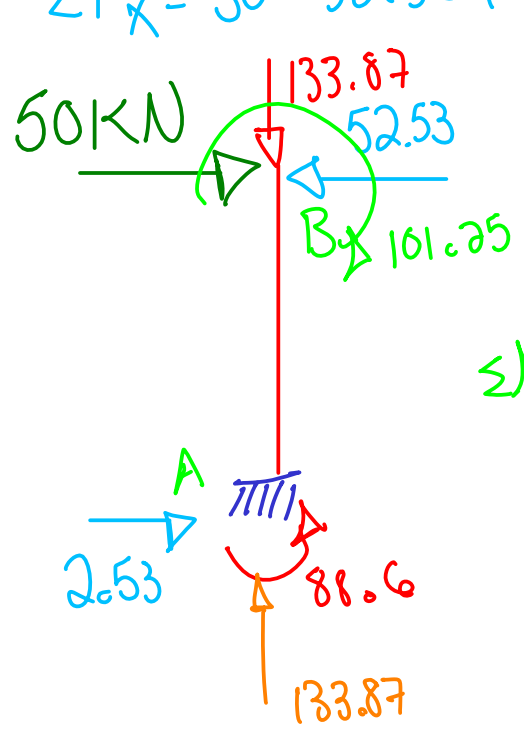
$$\sum M_A = 50(5) + 30(10)(5) - 166.14(10) + M_A = 0$$

$$M_A = -88.6 \text{ kN}\cdot\text{m}$$

$$\sum F_y = A_y - 30(10) + 166.14 = 0$$

$$A_y = 133.87 \text{ kN}$$

$$\sum F_x = 50 - 52.53 + A_x = 0 \therefore A_x = 2.53 \rightarrow$$



$$\sum M_B = 2.53(5) + 88.6 - M_B = 0$$

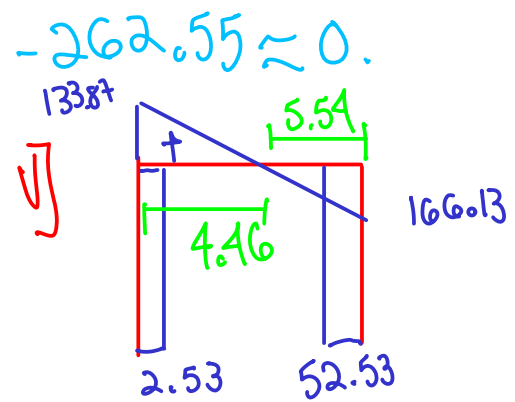
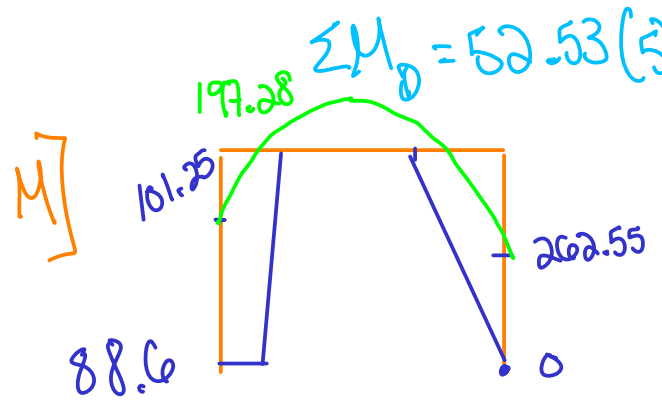
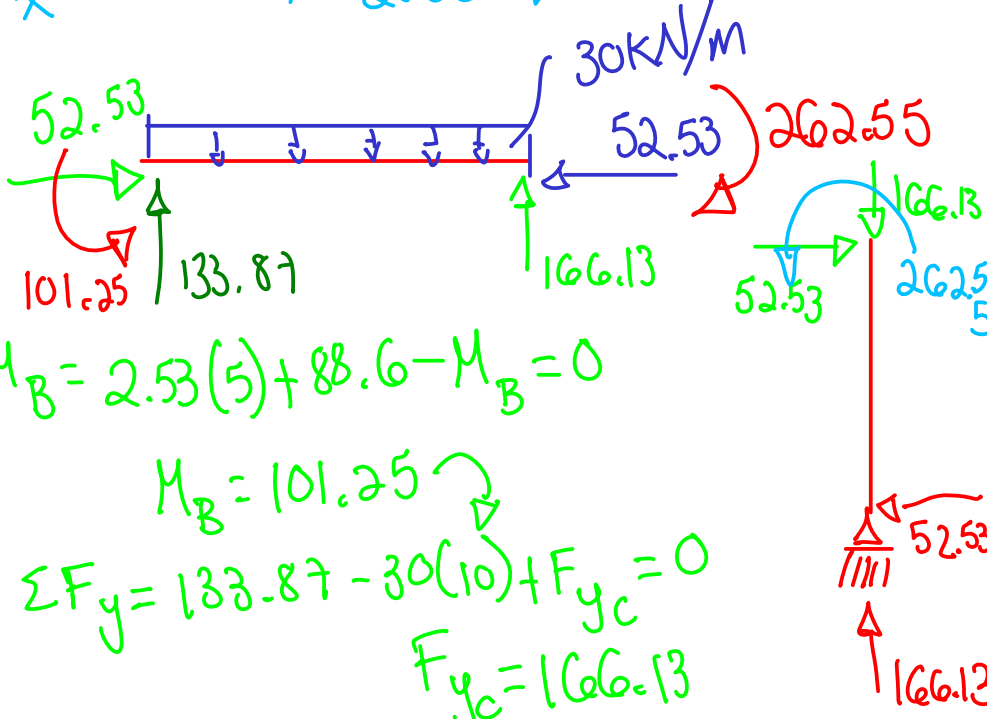
$$M_B = 101.25 \curvearrowright$$

$$\sum F_y = 133.87 - 30(10) + F_{y_c} = 0$$

$$F_{y_c} = 166.13$$

$$\sum M_C = M_C - 101.25 - 30(10)(5) + 133.87(10) = 0$$

$$M_C = 262.55$$

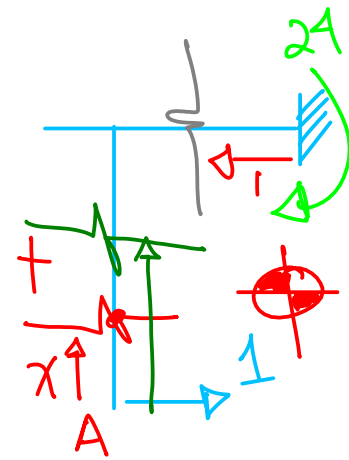
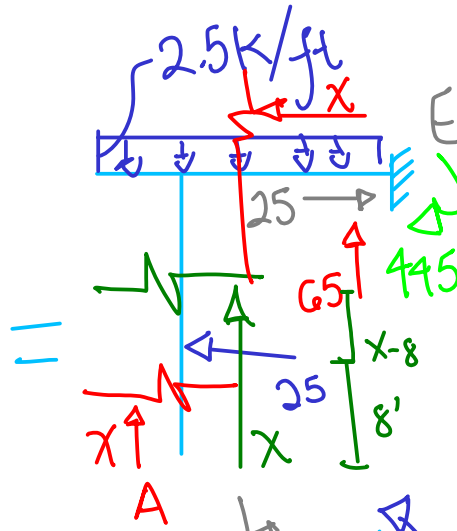
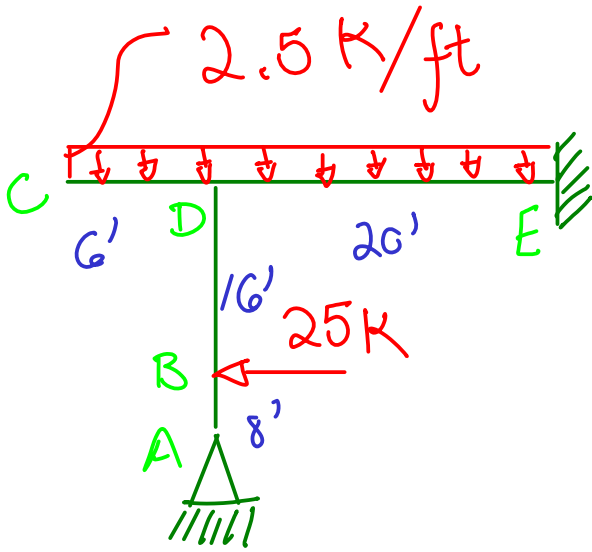
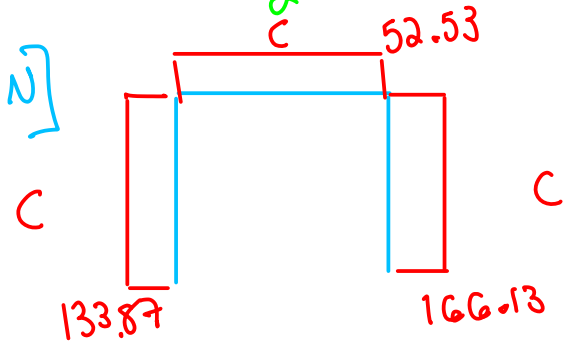


$$133.87 - 30x = 0$$

$$x = 4.46$$

$$-101.25 + \frac{1}{2} (4.46)(133.87) = 197.28$$

$$197.28 - \frac{1}{2} (5.54)(166.13) = -262.55$$



Marco original

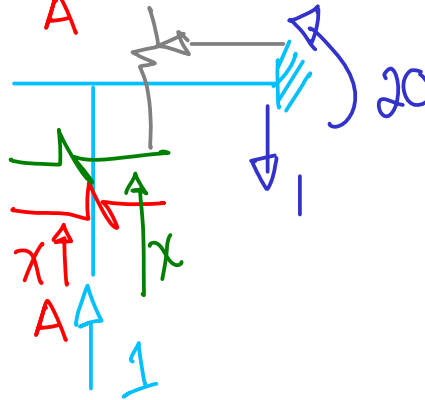
$$\sum M_E = (2.5)(20)\left(\frac{20}{2}\right) - 25(16) +$$

$$+ M_E = 0$$

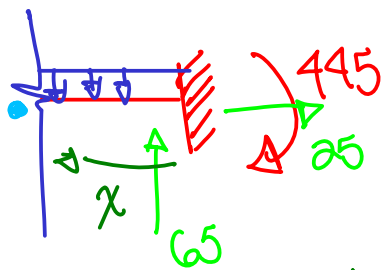
$$M_E = -445 \text{ ft}\cdot\text{k} = 445 \text{ ft}\cdot\text{k}$$

$$\sum F_y = 2.5(20) - E_y = 0$$

$$E_y = 65 \text{ k}$$



Segmento	Límites	Origen	M_0	M_x	M_y
AB	0-8	A	0	$-x$	0
BD	8-24	A	$25(x-8)$	$-x$	0
ED	0-20	E	$-445 - 2.5x(\frac{x}{2}) + 65x$	-24	$20-x$



$$\Delta_{Ax0} = \sum \int_0^x \frac{MM}{EI} dx = \int_0^8 0(-x) dx + \int_8^{24} [25(x-8)](-x) dx + \int_0^{20} (-445 - 1.25x^2 + 65x)(-24) dx = -\frac{78133.33}{EI}$$

$$\int_0^8 (0) \cdot (-x) dx + \int_8^{24} (25 \cdot (x-8)) \cdot (-x) dx + \int_0^{20} ((-1.25) \cdot x^2 + 65 \cdot x - 445) \cdot (-24) dx = -78133.3333$$

$$\Delta_{Ay0} = \int_0^8 (0) \cdot (0) dx + \int_8^{24} (25 \cdot (x-8)) \cdot (0) dx + \int_0^{20} ((-1.25) \cdot x^2 + 65 \cdot x - 445) \cdot (20-x) dx = -19000 = -\frac{19000}{EI}$$

$$\delta_{AxAy} = \int_0^8 (-x) \cdot (0) dx + \int_8^{24} (-x) \cdot (0) dx + \int_0^{20} (-24) \cdot (20-x) dx = -4800 = -\frac{4800}{EI}$$

$$\delta_{AyAx} =$$

$$\delta_{AyAy} = \int_0^8 (0) \cdot (0) dx + \int_8^{24} (0) \cdot (0) dx + \int_0^{20} (20-x) \cdot (20-x) dx = 2666.6667 = \frac{2666.67}{EI}$$

$$\delta_{A_x A_x} = \int_0^8 (-x) \cdot (-x) dx + \int_8^{24} (-x) \cdot (-x) dx + \int_0^{20} (-24) \cdot (-24) dx = 16128 = \frac{16128}{EI}$$

Ecuaciones de compatibilidad

$$\Delta_{A_x 0} + \delta_{A_x A_x} A_x + \delta_{A_x A_y} A_y = 0$$

$$\Delta_{A_y 0} + \delta_{A_y A_x} A_x + \delta_{A_y A_y} A_y = 0$$

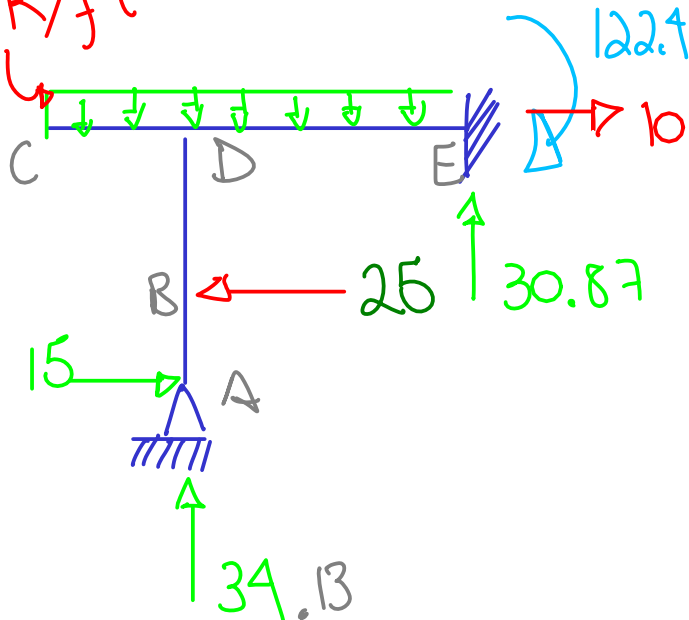
$$-78133 + 16128 A_x - 4800 A_y = 0$$

$$-19000 - 4800 A_x + 2666.67 A_y = 0$$

$$A_x = 15 \rightarrow$$

$$A_y = 34.13 \uparrow$$

2.5 K/ft



$$\sum M_E = M_E + (2.5)(26)(13)$$

$$-25(16) + 15(24)$$

$$-34.13(20) = 0$$

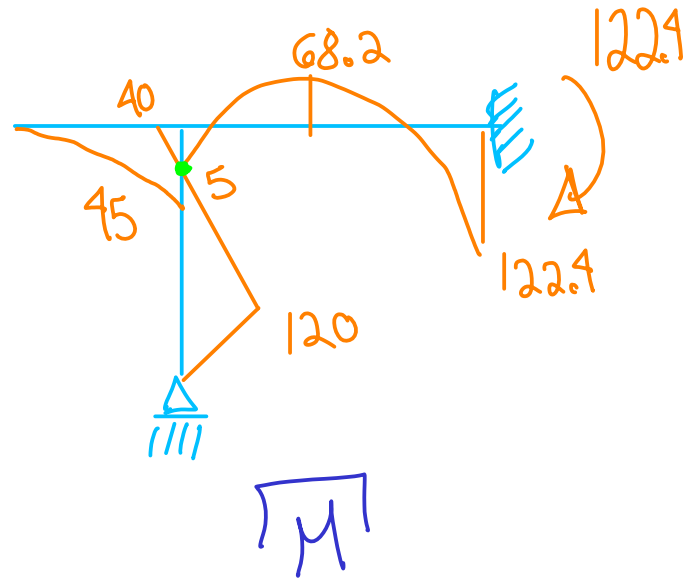
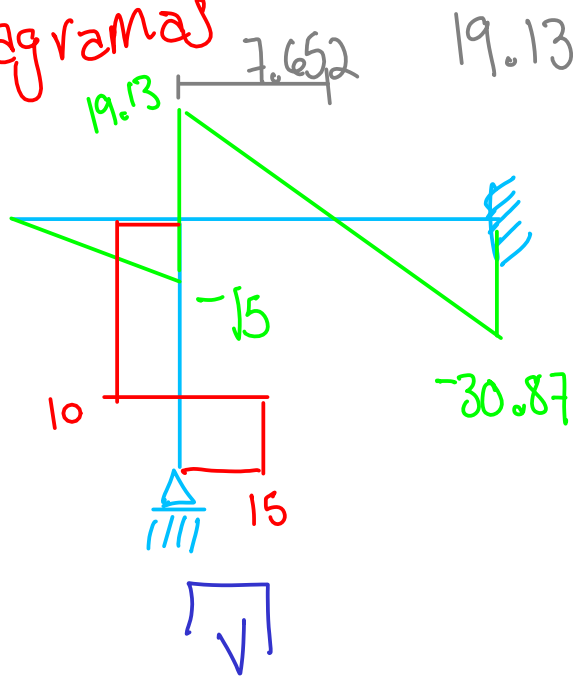
$$M_E = \underline{-122.4 \text{ (down)}}$$

$$M_E = 122.4 \text{ (up)}$$

$$\sum F_x = 15 - 25 + E_x = 0 \therefore E_x = 10 \rightarrow$$

$$\Sigma F_y = -2.5(20) + 34.13 + E_y = 0 \quad \therefore E_y = 30.87 \uparrow$$

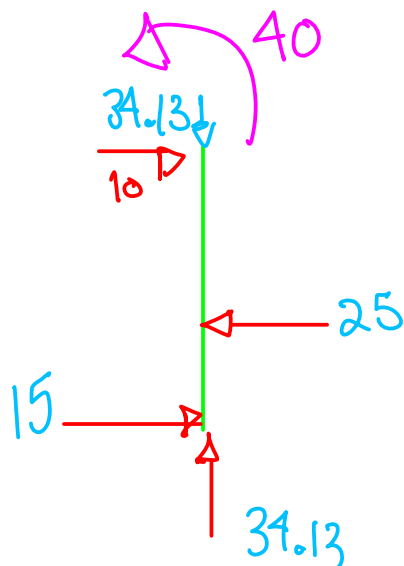
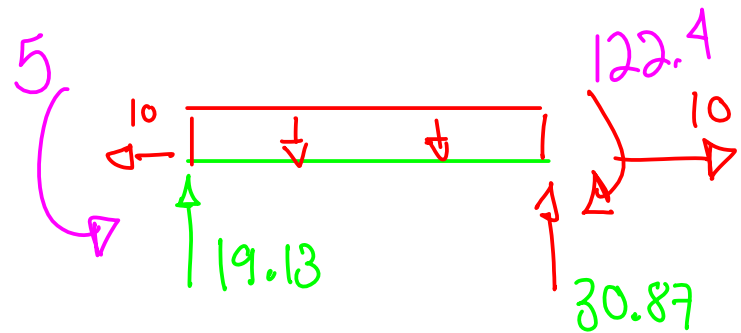
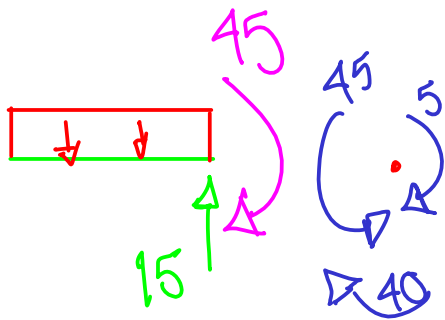
Diagramas



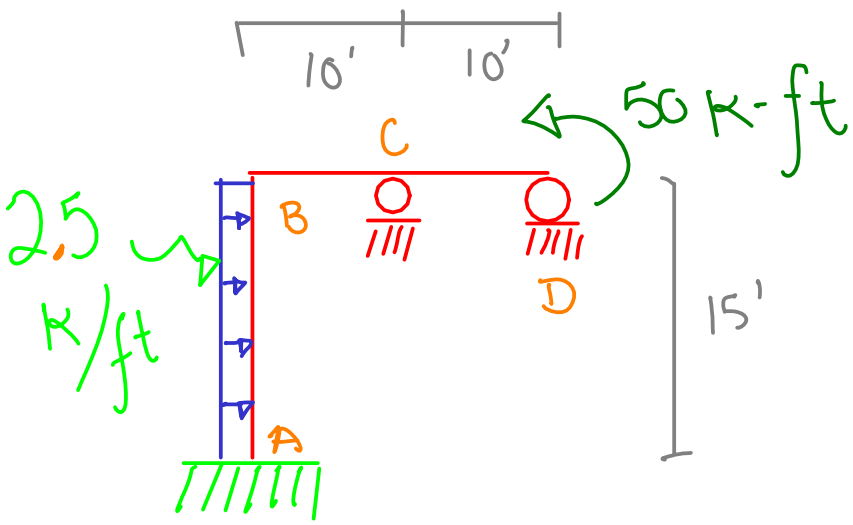
$$\frac{1}{2}(6)(15) = 45$$

$$\frac{1}{2}(30.87)(20 - 7.652) = 190.59$$

$$\frac{1}{2}(7.652)(19.13) = 73.2$$



$$\Sigma M = 2.5(20)(10) - 30.87(20) + 122.4 = 5$$



Este ejercicio lo puede consultar en el apartado debajo de esta página "Entradas históricas", debido a que se resolvió de manera manual en el pizarrón.