

$$I = \theta_B \text{ \& } \theta_D$$

$$M_{NF} = 2EK(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

$$M_{AB} = \frac{2E(120\text{in}^4)}{18 \cdot 12} (2(0) + \theta_B - 0) - \frac{2K}{12} (18\text{ft})^2 \cdot 12$$

$$M_{BA} = \frac{2E(120\text{in}^4)}{18 \cdot 12} (2\theta_B + 0 - 0) + \frac{2 \cdot 18^2 \cdot 12}{12}$$

$$M_{BD} = \frac{2E(60\text{in}^4)}{9 \cdot 12} (2\theta_B + \theta_D - 0) + 0$$

$$M_{DB} = \frac{2E(60)}{9 \cdot 12} (2\theta_D + \theta_B - 0) + 0$$

$$A_{NF} \cdot \bar{x}_{AB} = \frac{wl^4}{24}$$

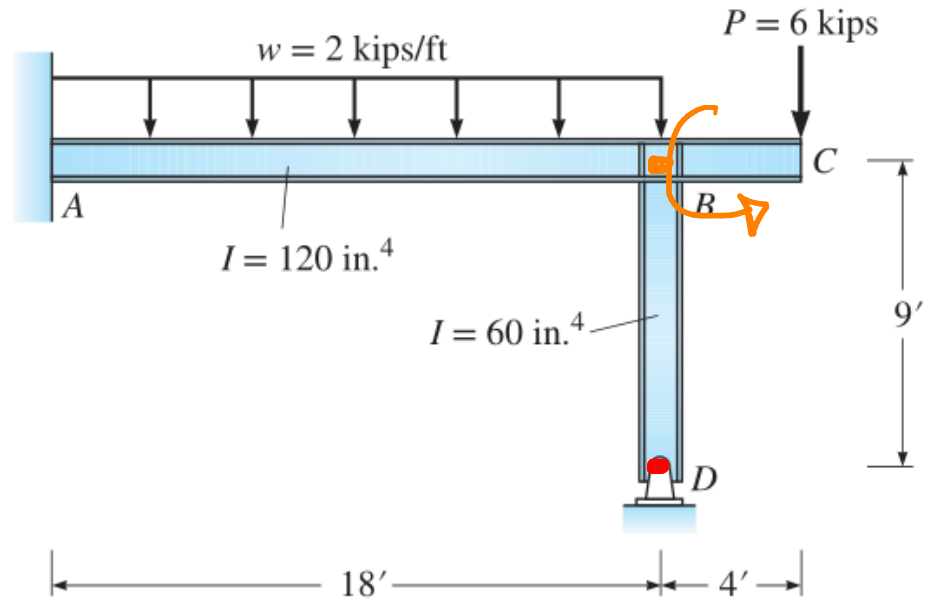
$$\frac{2}{24} \left(\frac{wl^4}{24} \right)^2 - \frac{4}{24} \left(\frac{wl^4}{24} \right)^2 = \frac{1}{12} wl^4 - \frac{1}{6} wl^4 = -\frac{1}{12} wl^4$$

$$M_{AB} = 1111E\theta_B - 648$$

$$M_{BA} = 2222E\theta_B + 648$$

$$M_{BD} = 2222E\theta_D + 1111E\theta_B$$

$$M_{DB} = 2222E\theta_D + 1111E\theta_B$$



$$\sum M_D = 0 = M_{DB}$$

$$\sum M_B = M_{BA} + M_{BD} - 24 = 0$$

$$(2222E\theta_B + 648) + (2222E\theta_D + 1111E\theta_B) - 288 = 0$$

$$2222E\theta_D + 1111E\theta_B = 0$$

$$1111E\theta_D + 4444E\theta_B = -360$$

$$\begin{bmatrix} 2.222 & 1.111 \\ 1.111 & 4.444 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -360 \end{bmatrix} = \begin{bmatrix} 46.2903 \\ -92.5807 \end{bmatrix}$$

$$\theta_D = \frac{46.29}{E}$$

$$\theta_B = \frac{-92.58}{E}$$

$$M_{AB} = 1111E \theta_B - 648$$

$$M_{BD} = 2222E \theta_B + 1111E \theta_D$$

$$M_{AB} = 1111E \left(\frac{-92.58}{E} \right) - 648$$

$$= -750.856 \text{ K}\cdot\text{in}$$

$$= -62.57 \text{ K}\cdot\text{ft}$$

$$M_{BD} = 2222E \left(\frac{-92.58}{E} \right) + 1111E \left(\frac{46.29}{E} \right)$$

$$M_{BD} = -154.284 \text{ K}\cdot\text{in}$$

$$= -12.857 \text{ K}\cdot\text{ft}$$

$$M_{BA} = 2222E \theta_B + 648$$

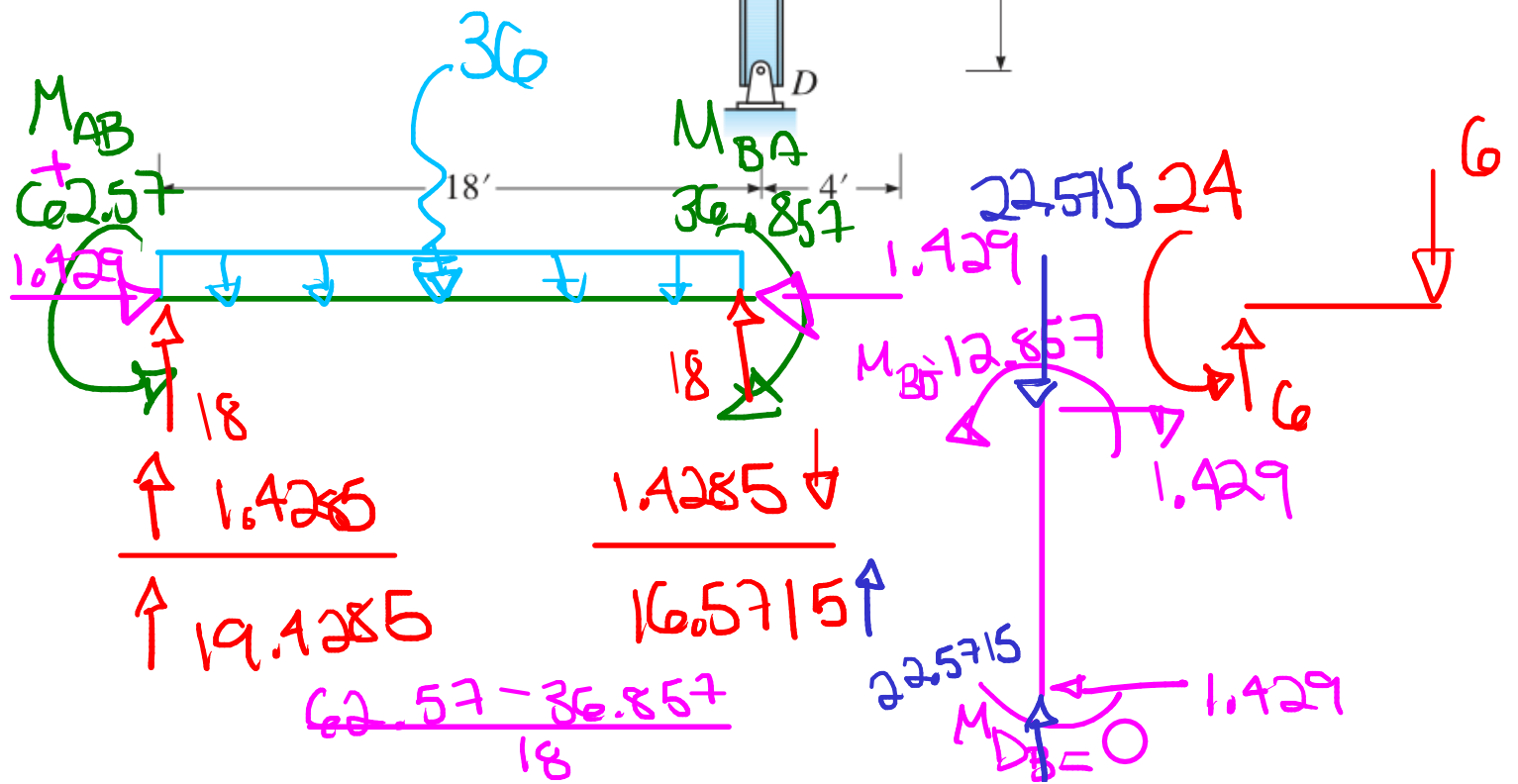
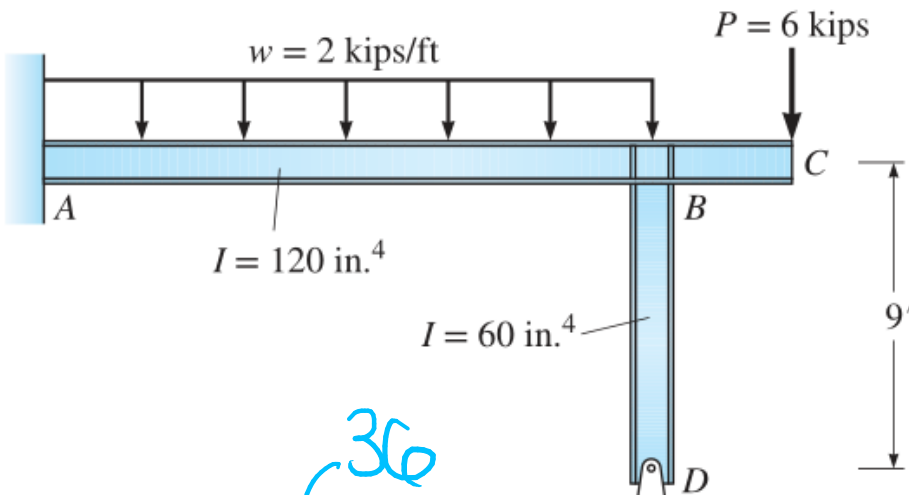
$$M_{DB} = 2222E \theta_D + 1111E \theta_B$$

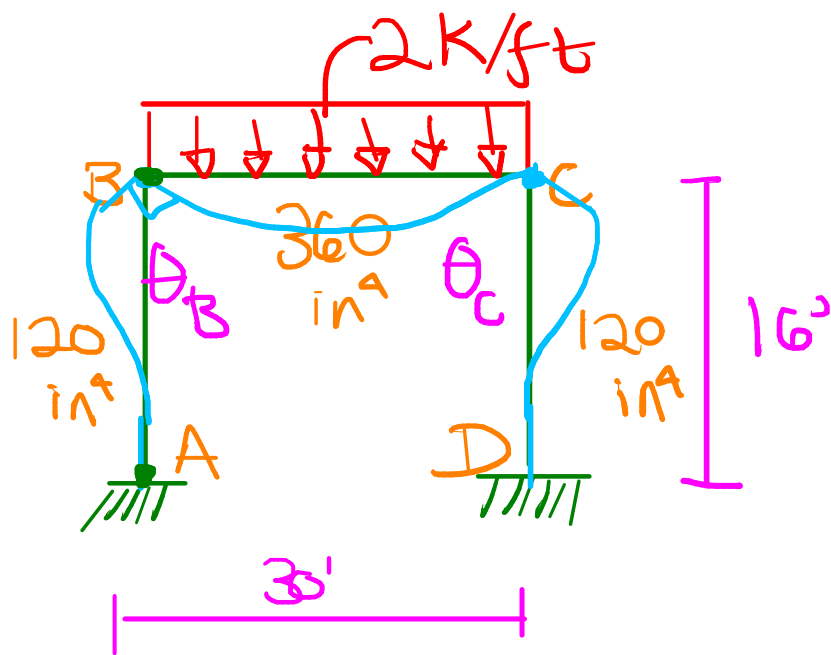
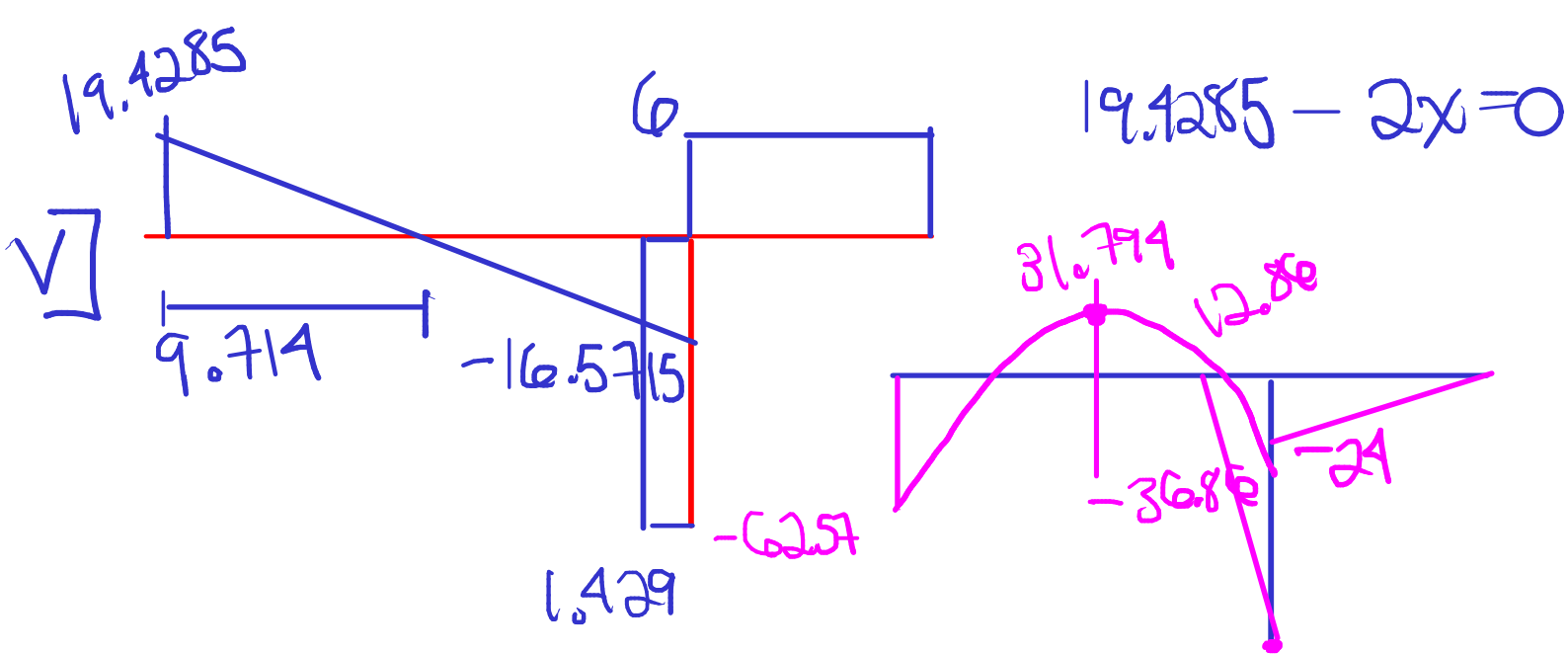
$$= 2222E \left(\frac{-92.58}{E} \right) + 648$$

$$= 442.29 \text{ K}\cdot\text{in} = 36.857 \text{ K}\cdot\text{ft}$$

$$M_{DB} = 2222E \left(\frac{46.29}{E} \right) + 1111E \left(\frac{-92.58}{E} \right)$$

$$= 0$$





$$\theta_B = -\theta_C$$

$$M_{NF} = 2EK(2\theta_N + \theta_F - 3\psi_{NF}) + FEM_{NF}$$

$$M_{AB} = \frac{2E(120)}{16 \cdot 12} (\theta_B) = 1.25E\theta_B$$

$$M_{BA} = \frac{2E(120)}{16 \cdot 12} (2\theta_B) = 2.50E\theta_B$$

$$M_{BC} = \frac{2E(360)}{30 \cdot 12} (2\theta_B + \theta_C) - \frac{(2 \frac{k}{ft})(30 \text{ ft})^2 \cdot 12}{12}$$

$$= 2E(2\theta - \theta) - 1800 = 2E\theta - 1800$$

$$\sum M_B = 0 = M_{BA} + M_{BC} = 0$$

$$2.5E\theta + 2E\theta - 1800 = 0$$

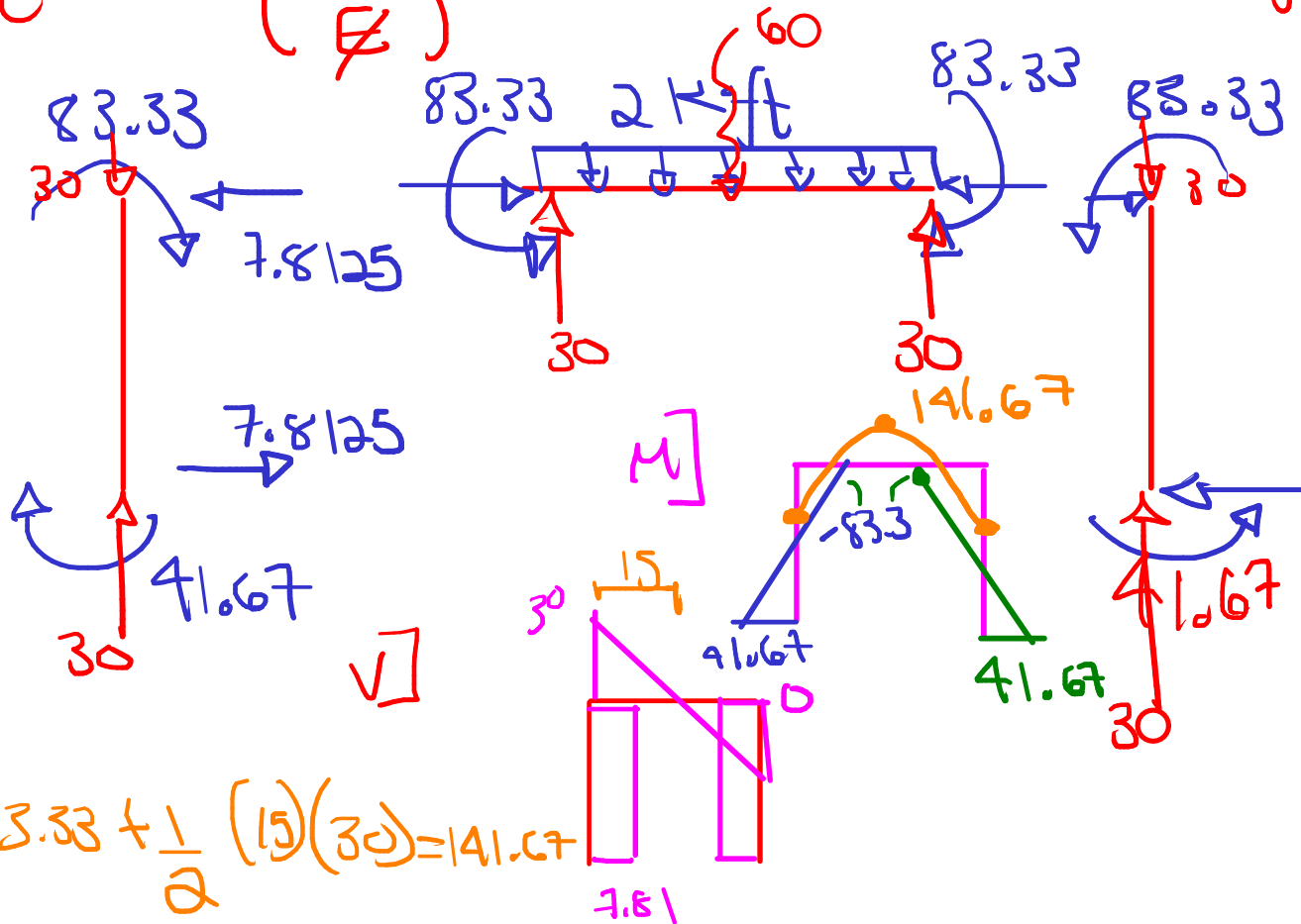
$$4.5E\theta = 1800$$

$$\theta = \frac{1800}{4.5E} = \frac{400}{E}$$

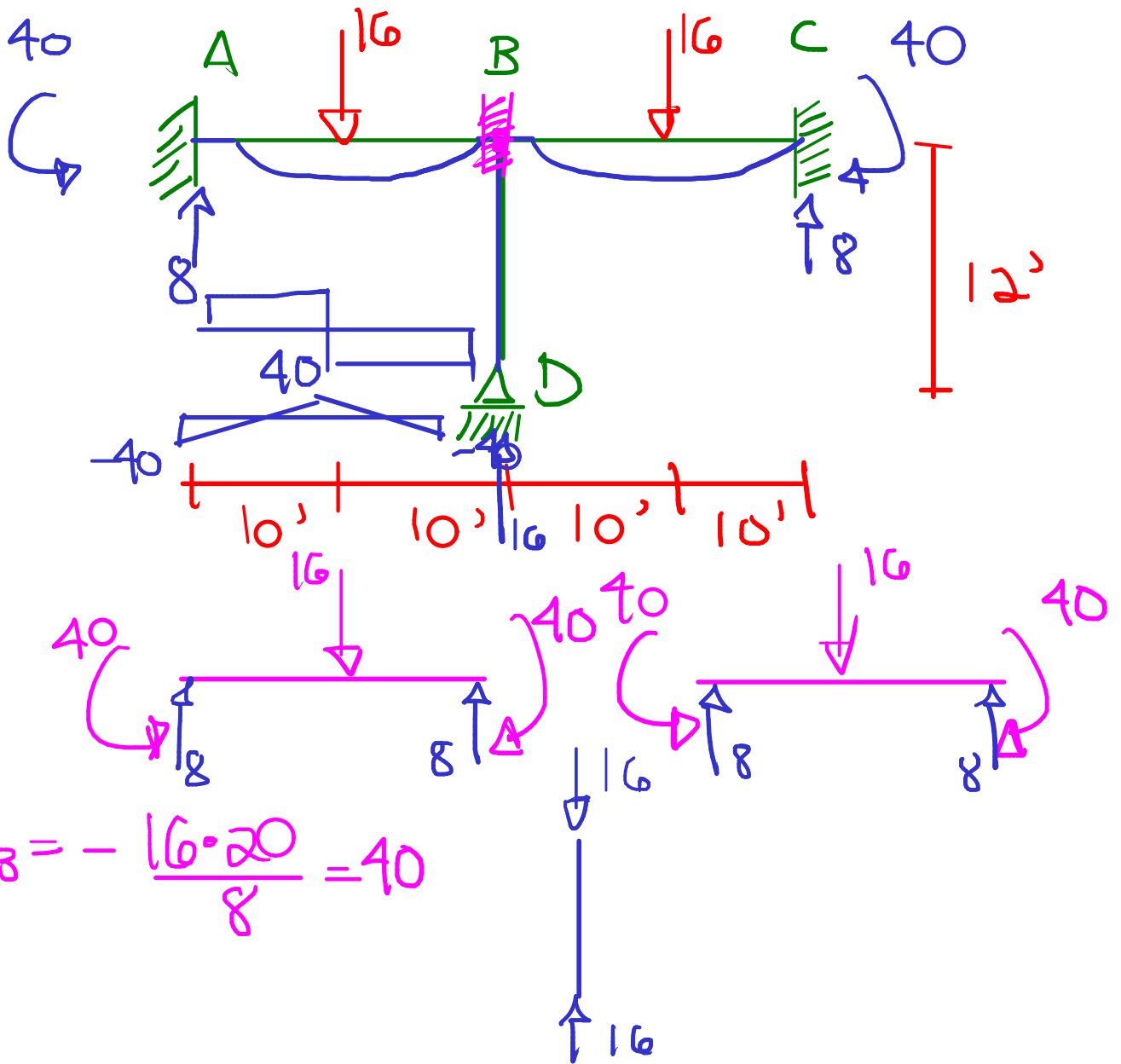
$$M_{AB} = 1.25E \left(\frac{400}{E} \right) = 500 \text{ Kip}\cdot\text{in} = 41.67 \text{ K}\cdot\text{ft}$$

$$M_{BA} = 2.50E \left(\frac{400}{E} \right) = 1000 \text{ K}\cdot\text{in} = 83.33 \text{ K}\cdot\text{ft}$$

$$M_{BC} = 2E \left(\frac{400}{E} \right) - 1800 = -1000 \text{ K}\cdot\text{in} = -83.33 \text{ K}\cdot\text{ft}$$

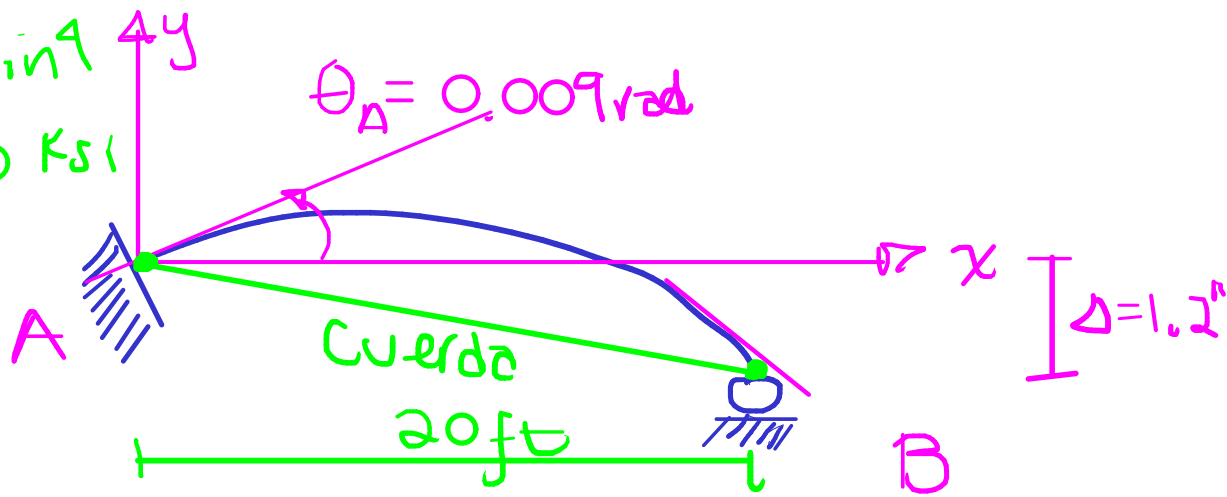


$$M_{NF} = 2EK(2\theta_N + \theta_F - 3\psi_{NF}) + FEM_{NF}$$



$$I = 360 \text{ in}^4$$

$$E = 29000 \text{ Ksi}$$



$$M_{NF} = 2EK(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

$$\psi_{AB} = \frac{1.2 \text{ in}}{20 \cdot 12 \text{ in}} = 0.005 \text{ rad}$$

$$M_{AB} = \frac{2E(360)}{20 \cdot 12} (2(-0.009) + \theta_B - 3(0.005)) + 0$$

$$M_{BA} = \frac{2E(360)}{20 \cdot 12} (2\theta_B - 0.009 - 3(0.005)) + 0$$

$$\sum M_B = 0 = M_{BA}$$

$$3E(2\theta_B - 0.024) = 0$$

$$6E\theta_B - 0.072E = 0$$

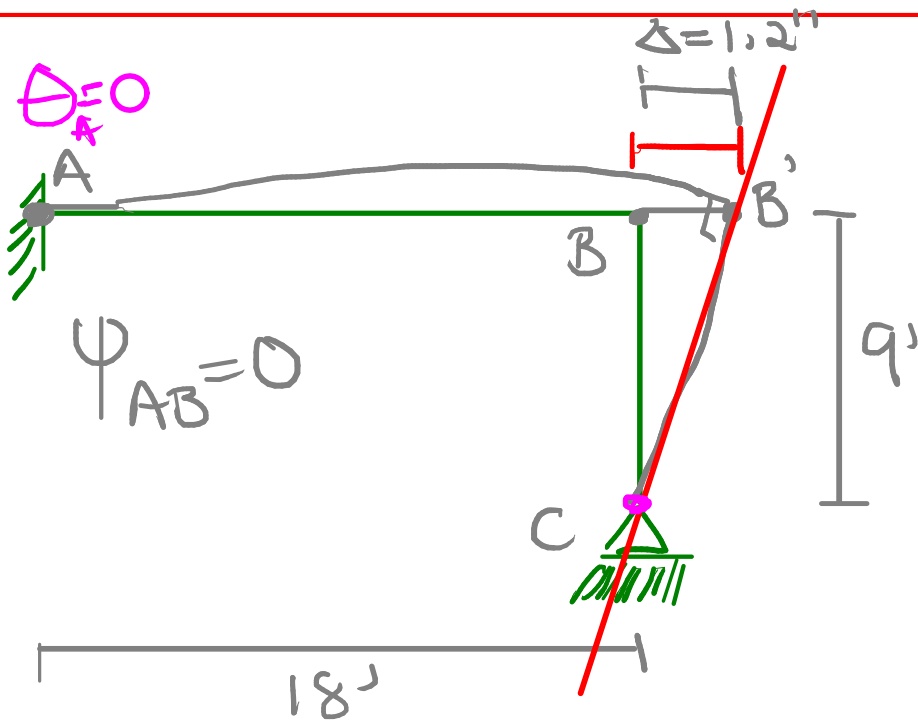
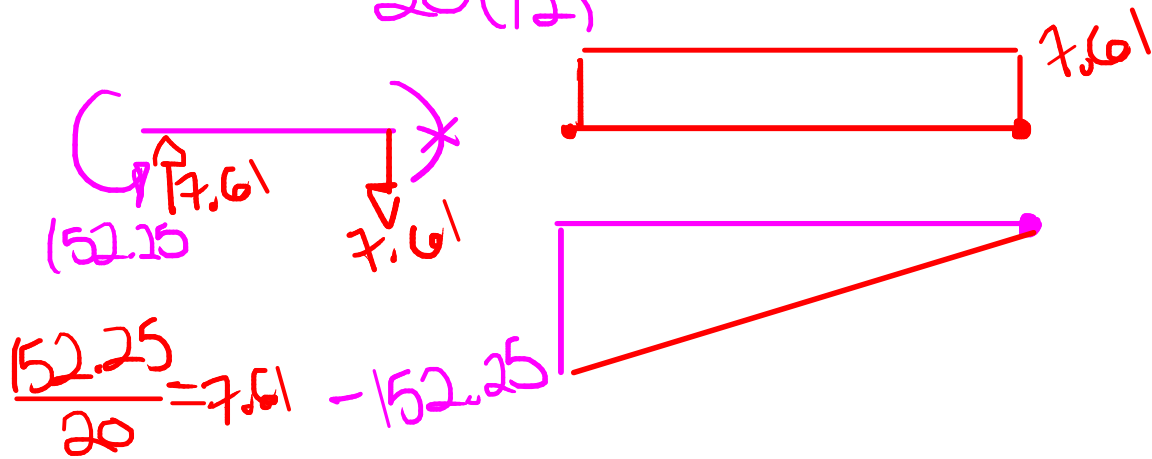
$$6E\theta_B = 0.072E$$

$$\theta_B = \frac{0.072E}{6E} = 0.012 \text{ rad}$$

$$M_{AB} = \frac{2(29000)(360)}{20 \cdot 12} (2(-0.009) + 0.012 - 3(0.005))$$

$$= -1827 \text{ Kip} \cdot \text{in} = -152.25 \text{ Kip} \cdot \text{ft}$$

$$M_{BA} = \frac{2(29000)(360)}{20(12)} (2 \cdot 0.012 - 0.009 - 3(0.005)) = 0$$



$$I = 240 \text{ in}^4$$

$$E = 29000 \text{ ksi}$$

$$\psi_{BC} = \frac{1.2}{9 \times 12} = \frac{1}{90} \text{ rad}$$

$$M_{NF} = 2EK(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

$$M_{AB} = \frac{2E(240)}{18 \times 12} (2(0) + \theta_B - 3(0)) + 0$$

$$= 2.2222 E \theta_B$$

$$M_{BA} = \frac{2E(240)}{18 \times 12} (2\theta_B + 0 - 3(0)) + 0$$

$$= 4.4444 E \theta_B$$

$$M_{BC} = \frac{2E(240)}{9 \times 12} (2\theta_B + \theta_C - 3(\frac{1}{90})) + 0$$

$$= 8.8889 \theta_B + 4.4444 \theta_C - 0.1481 E$$

$$M_{CB} = \frac{2E(240)}{9 \times 12} (2\theta_C + \theta_B - 3\left(\frac{1}{90}\right)) + 0$$

$$= 8.8889\theta_C E + 4.4444\theta_B E - 0.1481E$$

$$\textcircled{1} \sum M_C = M_{CB} = 0$$

$$\textcircled{2} \sum M_B = M_{BA} + M_{BC} = 0$$

$$\textcircled{1} 8.8889\theta_C E + 4.4444\theta_B E - 0.1481E = 0$$

$$\textcircled{2} 4.4444E\theta_B + 8.8889E\theta_B + 4.4444E\theta_C - 0.1481E = 0$$

$$\textcircled{2} 13.3333E\theta_B + 4.4444E\theta_C - 0.1481E = 0$$

$$\begin{bmatrix} 13.3333 & 4.4444 \\ 4.4444 & 8.8889 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.1481 \\ 0.1481 \end{bmatrix} = \begin{bmatrix} 0.0067 \\ 0.0133 \end{bmatrix} \begin{matrix} \theta_B \\ \theta_C \end{matrix}$$

$$\theta_B := 0.006665 \quad E := 29000 \text{ ksi}$$

$$\theta_C := 0.013329$$

$$M_{AB} := 2.2222 \text{ in}^3 \cdot E \cdot \theta_B = 429.5179 \text{ kip in}$$

$$M_{AB} = 35.7932 \text{ kip ft}$$

$$M_{BA} := 4.4444 \text{ in}^3 \cdot E \cdot \theta_B = 859.0359 \text{ kip in}$$

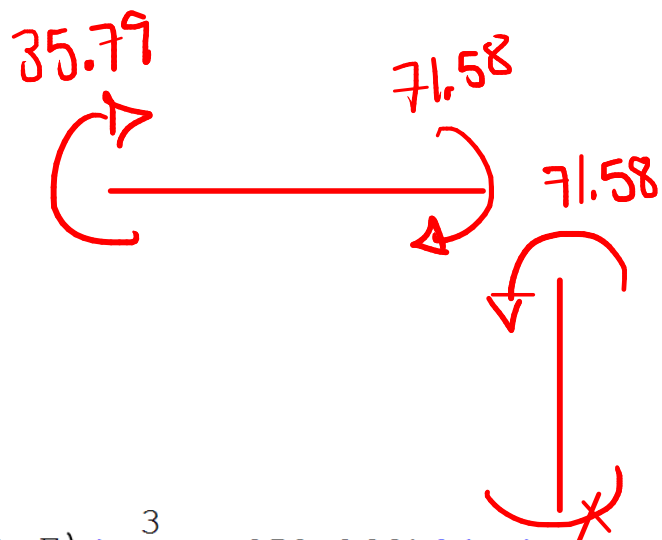
$$M_{BA} = 71.5863 \text{ kip ft}$$

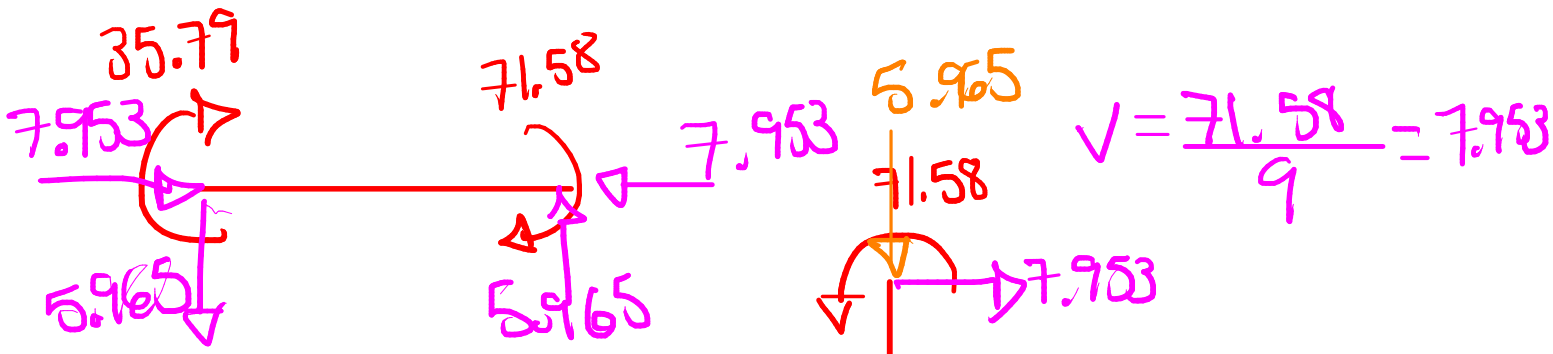
$$M_{BC} := (8.8889 \cdot \theta_B \cdot E + 4.4444 \cdot \theta_C \cdot E - 0.1481 \cdot E) \text{ in}^3 = -858.8661 \text{ kip in}$$

$$M_{BC} = -71.5722 \text{ kip ft}$$

$$M_{CB} := (8.8889 \cdot \theta_C \cdot E + 4.4444 \cdot \theta_B \cdot E - 0.1481 \cdot E) \text{ in}^3 = 0.0601 \text{ kip in}$$

$$M_{CB} = 0.005 \text{ kip ft}$$





$$V = \frac{35.79 + 71.58}{18} = 5.965$$

$$V = \frac{71.58}{9} = 7.953$$

