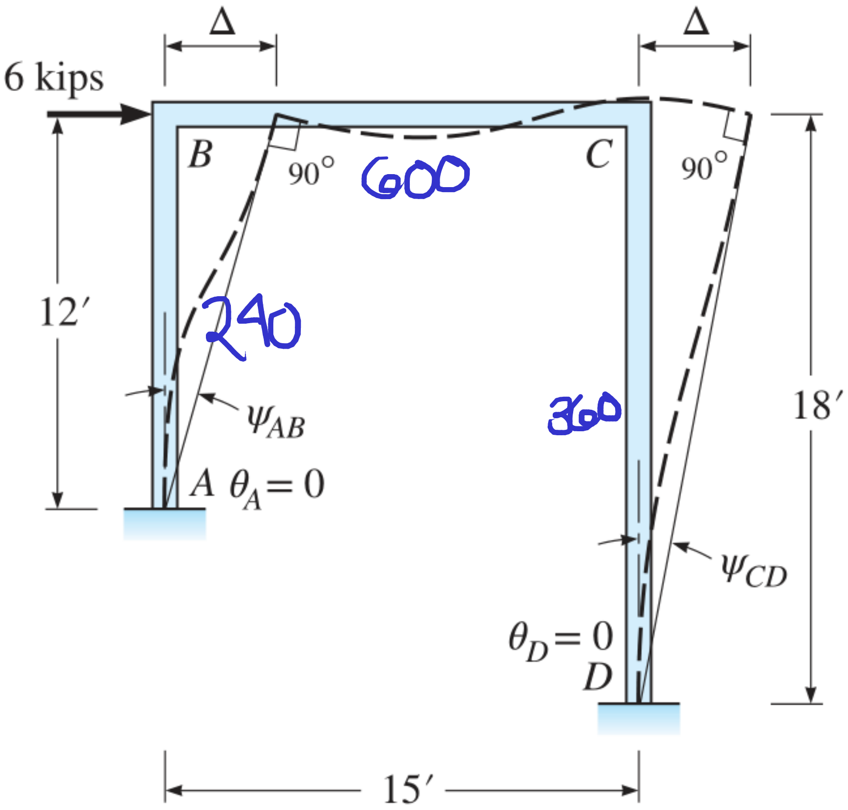
$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \cancel{FEM_{NF}}$$

$4K$



$$\psi_{AB} = \frac{\Delta}{h} = \frac{\Delta}{12}$$

$$\psi_{CD} = \frac{\Delta}{18}$$

$$12\psi_{AB} = \Delta$$

$$18\psi_{CD} = \Delta$$

$$12\psi_{AB} = 18\psi_{CD}$$

$$\psi_{AB} = \frac{18}{12}\psi_{CD}$$

$$\psi_{AB} = 1.5\psi_{CD}$$

$$K_{AB} = \frac{E(240)}{12 \cdot 12} = \frac{5}{3}E$$

$$K_{BC} = \frac{E(600)}{15 \cdot 12} = \frac{10}{3}E$$

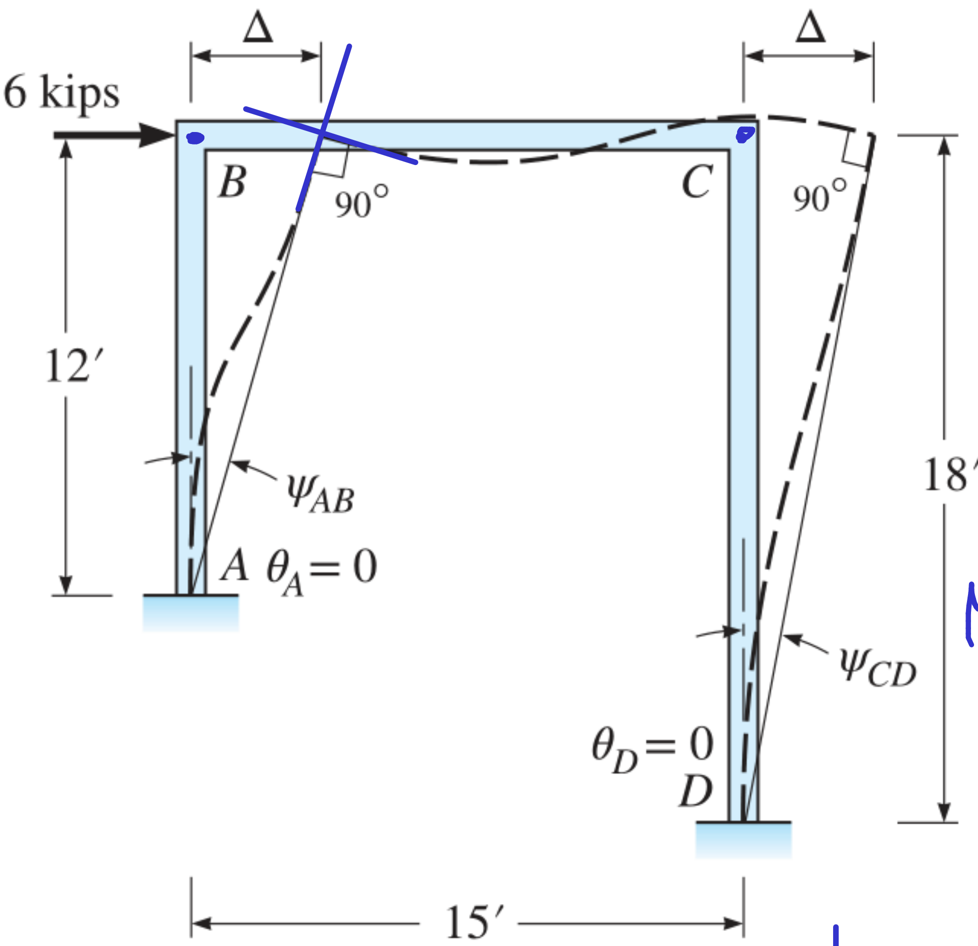
$$K_{CD} = \frac{E(360)}{18 \cdot 12} = \frac{5}{3}E$$

Sea $\overset{KE}{\frac{5}{3}E} = K$

$K_{AB} = K$ $\quad K_{BC} = 2K$ $\quad K_{CD} = K$

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

ecr ∂r



$M_{CD} = 2K(2\theta_C - 3\psi_{CD})$

$M_{DC} = 2K(\theta_C - 3\psi_{CD})$

$\psi_{AB} = 1.5\psi_{CD}$

$M_{AB} = 2K(\theta_B - 3\psi_{AB})$

$M_{AB} = 2K(\theta_B - 4.5\psi_{CD})$

$M_{BA} = 2K(2\theta_B - 3\psi_{AB})$

$M_{BA} = 2K(2\theta_B - 4.5\psi_{CD})$

$M_{BC} = 4K(2\theta_B + \theta_C)$

$M_{CB} = 4K(2\theta_C + \theta_B)$

Nodo B → $\Sigma M_B = 0 = M_{BA} + M_{BC} = 0$

Nodo C → $\Sigma M_C = 0 = M_{CB} + M_{CD} = 0$

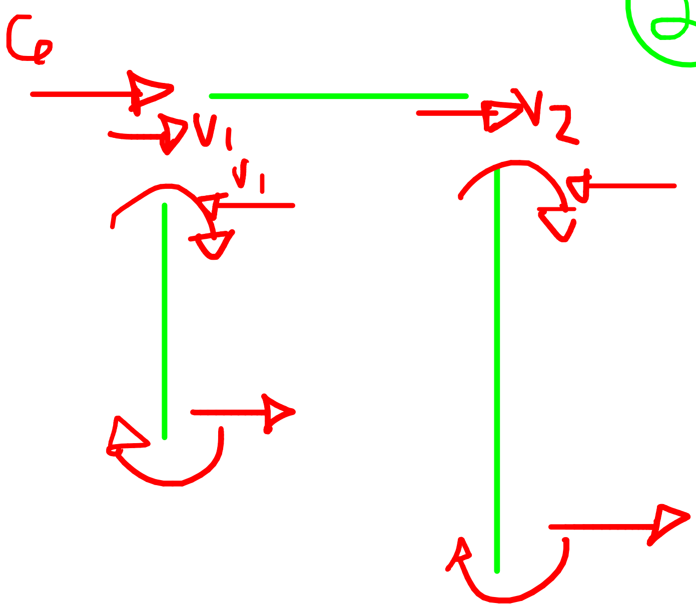
$4K\theta_B - 9K\psi_{CD} + 8K\theta_B + 4K\theta_C = 0$

$12K\theta_B + 4K\theta_C - 9K\psi_{CD} = 0$

1 $\quad 12\theta_B + 4\theta_C - 9\psi_{CD} = 0$

$$8\cancel{K}\theta_C + 4\cancel{K}\theta_B + 4\cancel{K}\theta_C - 6\cancel{K}\psi_{CD} = 0$$

(2) $$4\theta_B + 12\theta_C - 6\psi_{CD} = 0$$



(3) $$V_1 + V_2 + Q = 0$$

$$\frac{M_{AB}+M_{BA}}{12} + \frac{M_{CD}+M_{DC}}{18} + 6 =$$

$$\frac{2K\cdot\theta_B - 9\cdot K\cdot\psi_{CD} + 4\cdot K\cdot\theta_B - 9\cdot K\cdot\psi_{CD}}{12} + \frac{4\cdot K\cdot\theta_C - 6\cdot K\cdot\psi_{CD} + 2\cdot K\cdot\theta_C - 6\cdot K\cdot\psi_{CD}}{18} + 6 = 0$$

$$\frac{6\cdot K\cdot\theta_B - 18\cdot K\cdot\psi_{CD}}{12} + \frac{6\cdot K\cdot\theta_C - 12\cdot K\cdot\psi_{CD}}{18} + 6 = 0$$

$$\frac{6\cdot K\cdot\theta_B - 18\cdot K\cdot\psi_{CD}}{12} + \frac{6\cdot K\cdot\theta_C - 12\cdot K\cdot\psi_{CD}}{18} = -6$$

$$\frac{9\cdot K\cdot\theta_B - 27\cdot K\cdot\psi_{CD} + 6\cdot K\cdot\theta_C - 12\cdot K\cdot\psi_{CD}}{18} = -6$$

(3) $$9\cdot\theta_B + 6\cdot\theta_C - 39\cdot\psi_{CD} = \frac{-108}{K}$$

$$\begin{pmatrix} 12 & 4 & -9 \\ 4 & 12 & -6 \\ 9 & 6 & -39 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ \frac{-108}{K} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{756}{335\cdot K} \\ \frac{324}{335\cdot K} \\ \frac{1152}{335\cdot K} \end{pmatrix} \begin{matrix} = \theta_B \\ = \theta_C \\ = \psi_{CD} \end{matrix}$$

$$M_{AB} := 2\cdot K\cdot\left(\frac{756}{335\cdot K} - 4.5\cdot\frac{1152}{335\cdot K}\right) = -26.436 \quad k\cdot ft$$

$$M_{BA} := 2\cdot K\cdot\left(2\cdot\frac{756}{335\cdot K} - 4.5\cdot\frac{1152}{335\cdot K}\right) = -21.922$$

$$M_{BC} := 4\cdot K\cdot\left(2\cdot\frac{756}{335\cdot K} + \frac{324}{335\cdot K}\right) = 21.922$$

−2.579

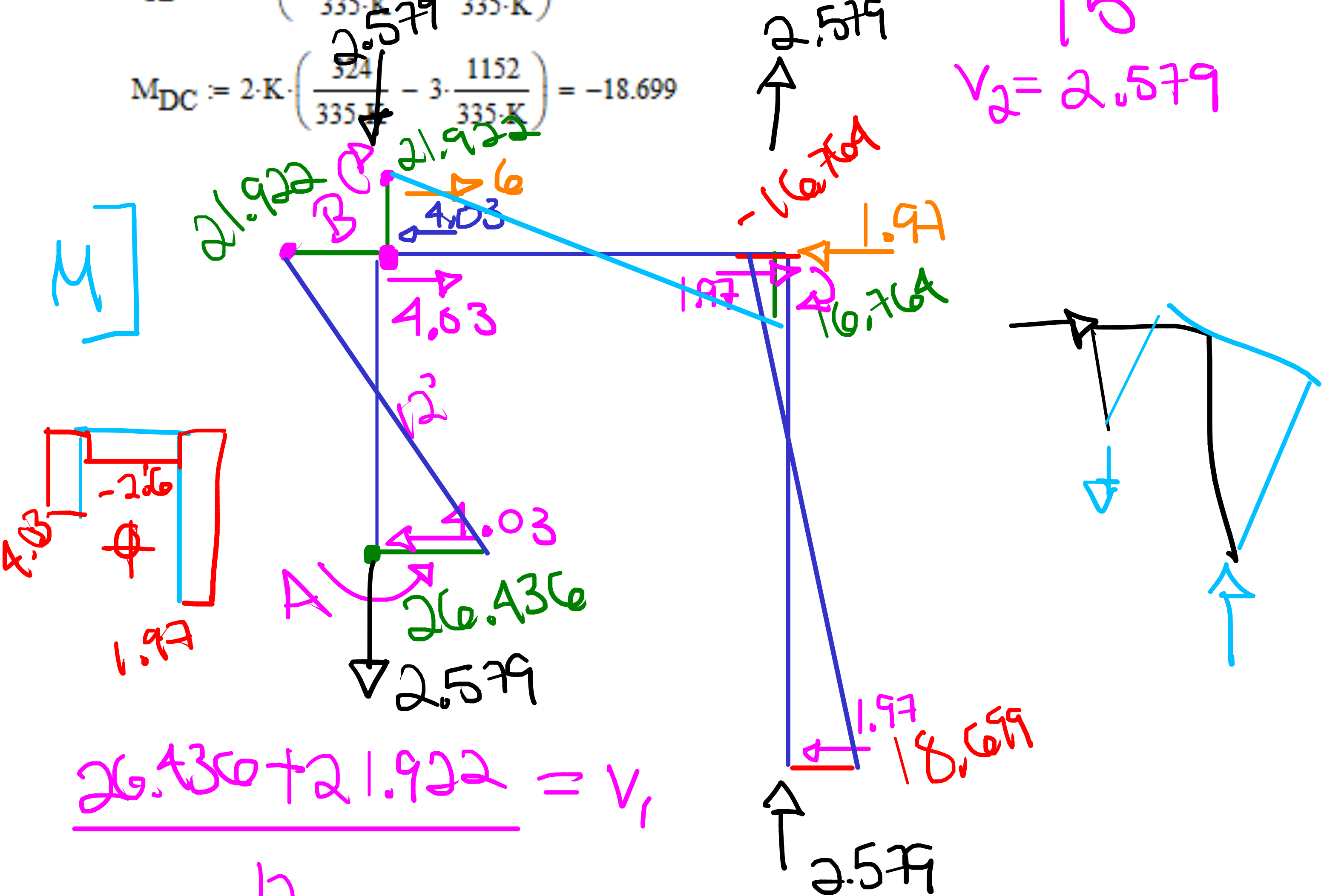
$$M_{CB} := 4\cdot K\cdot\left(2\cdot\frac{324}{335\cdot K} + \frac{756}{335\cdot K}\right) = 16.764$$

$$M_{CD} := 2\cdot K\cdot\left(2\cdot\frac{324}{335\cdot K} - 3\cdot\frac{1152}{335\cdot K}\right) = -16.764$$

$$M_{DC} := 2\cdot K\cdot\left(\frac{324}{335\cdot K} - 3\cdot\frac{1152}{335\cdot K}\right) = -18.699$$

$$V_2 = \frac{21.922 + 16.764}{15}$$

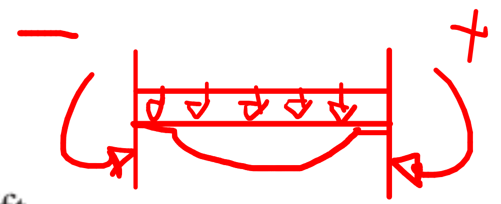
$$V_2 = 2.579$$

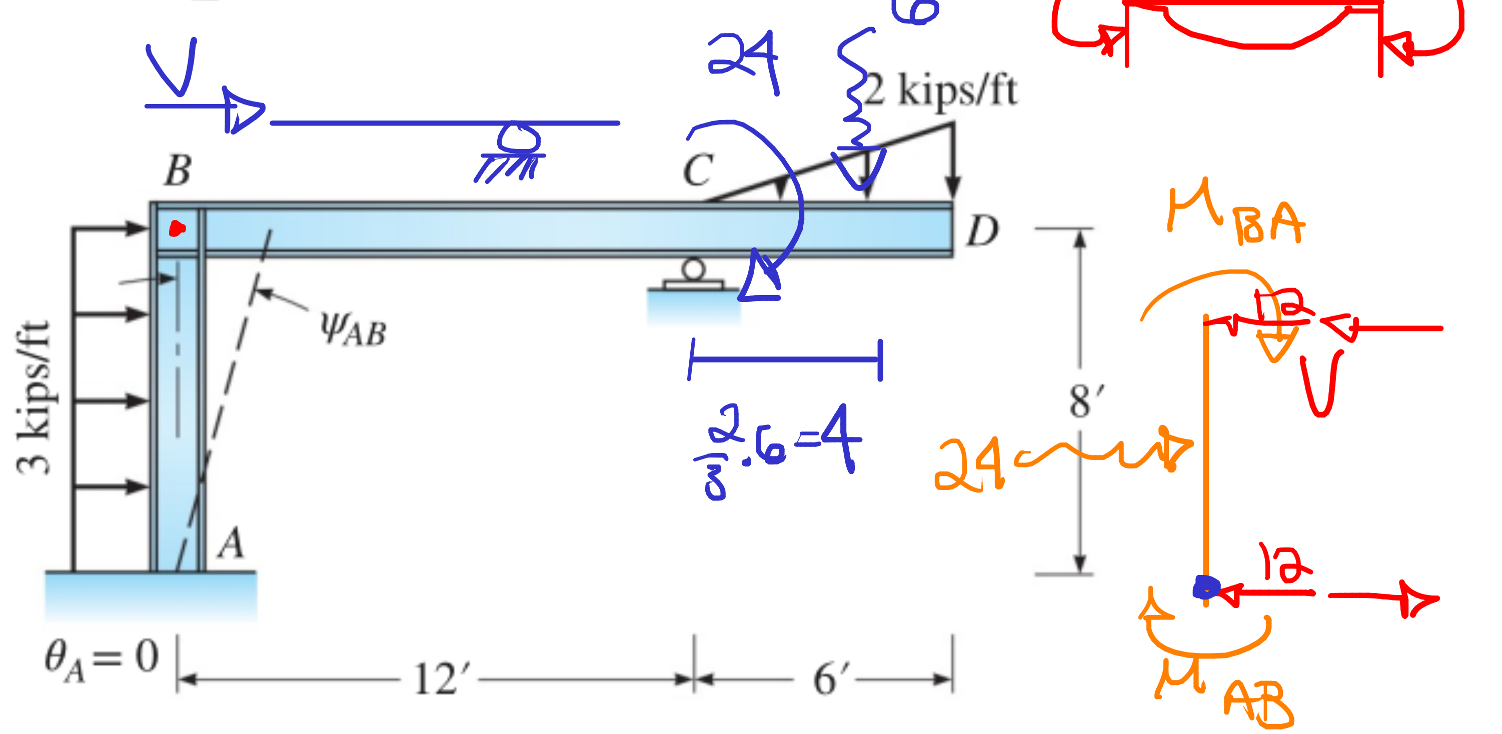


$$\frac{26.436 + 21.922}{12} = V_1$$

$$V_1 = 4.03$$

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$





$$M_{AB} = \frac{2EI}{8}(\theta_B - 3\psi_{AB}) - \frac{3(8)^2}{12}$$

$$M_{BA} = \frac{2EI}{8}(2\theta_B - 3\psi_{AB}) + \frac{3(8)^2}{12}$$

$$\Sigma M_A = M_{AB} + M_{BA} + 24(4) - V(8) = 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C)$$

$$V = \frac{M_{AB} + M_{BA} + 96}{8}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B)$$

$$\Sigma M_B = M_{BA} + M_{BC} = \frac{4}{8}EI\theta_B - \frac{6}{8}EI\psi_{AB} + 16 + \frac{4}{12}EI\theta_B + \frac{2}{12}EI\theta_C = 0$$

$$\frac{5}{6}EI\theta_B + \frac{1}{6}EI\theta_C - \frac{3}{4}EI\psi_{AB} = -16$$

$$\frac{10EI\theta_B + 2EI\theta_C - 9EI\psi_{AB}}{12} = -16$$

$$10\theta_B + 2\theta_C - 9\psi_{AB} = \frac{-192}{EI}$$

$$\Sigma M_C = M_{CB} - 24 = 0$$

$$\frac{2}{6}EI\theta_C + \frac{1}{6}EI\theta_B - 24 = 0$$

$$2\theta_C + \theta_B = \frac{24(6)}{EI}$$

$$\theta_B + 2\theta_C = \frac{144}{EI}$$

$$\Sigma F_x = V = 0$$

$$\frac{M_{AB} + M_{BA} + 96}{8} = 0$$

$$M_{AB} + M_{BA} = -96$$

$$M_{AB} = \frac{2EI}{8}(\theta_B - 3\psi_{AB}) - \frac{3(8)^2}{12}$$

$$M_{BA} = \frac{2EI}{8}(2\theta_B - 3\psi_{AB}) + \frac{3(8)^2}{12}$$

$$\frac{2}{8}EI\theta_B - \frac{6}{8}EI\psi_{AB} \cancel{-16}$$

$$+\frac{4}{8}EI\theta_B - \frac{6}{8}EI\psi_{AB} \cancel{+16} = -96$$

$$\frac{6}{8}EI\theta_B - \frac{12}{8}EI\psi_{AB} = -96$$

$$6EI\theta_B - 12EI\psi_{AB} = -768$$

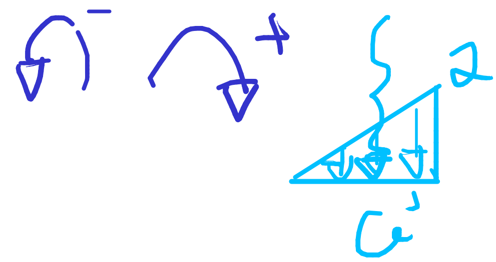
$$6\theta_B - 12\psi_{AB} = \frac{-768}{EI}$$

$$3\theta_B - 6\psi_{AB} = \frac{-384}{EI}$$

$$\theta_B - 2\psi_{AB} = \frac{-128}{EI}$$

$$\begin{array}{ccc} \theta_B & \theta_C & \psi_{AB} \end{array}$$

$$\begin{bmatrix} 10 & 2 & -9 \\ 1 & 2 & 0 \\ 1 & 0 & -2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -192 \\ 144 \\ -128 \end{bmatrix} = \begin{bmatrix} 53.3333 \\ 45.3333 \\ 90.6667 \end{bmatrix} \begin{array}{l} \theta_B \\ \theta_C \\ \psi_{AB} \end{array}$$

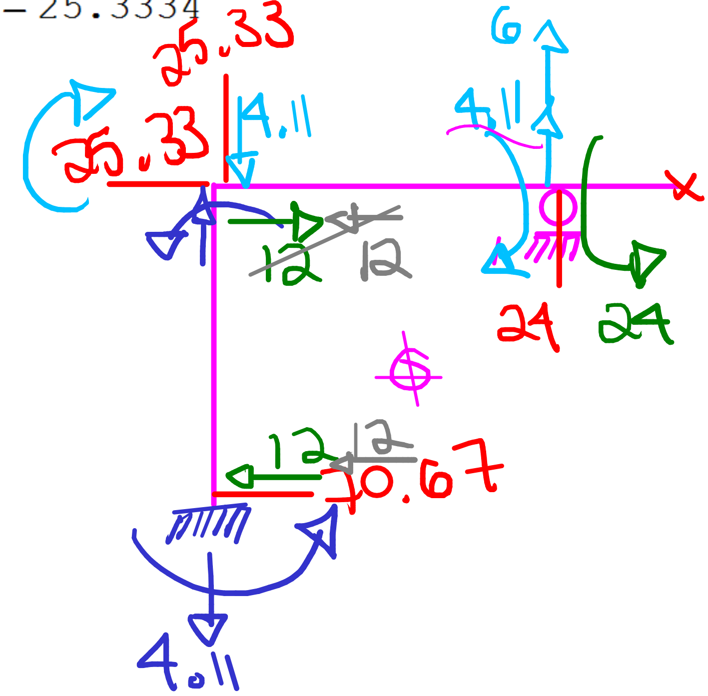


$$M_{AB} := \frac{2}{8} \cdot \left(\theta_B - 3 \cdot \psi_{AB}\right) - \frac{3 \cdot (8)^2}{12} = -70.6667$$

$$M_{BA} := \frac{2}{8} \cdot \left(2 \cdot \theta_B - 3 \cdot \psi_{AB}\right) + \frac{3 \cdot (8)^2}{12} = -25.3334$$

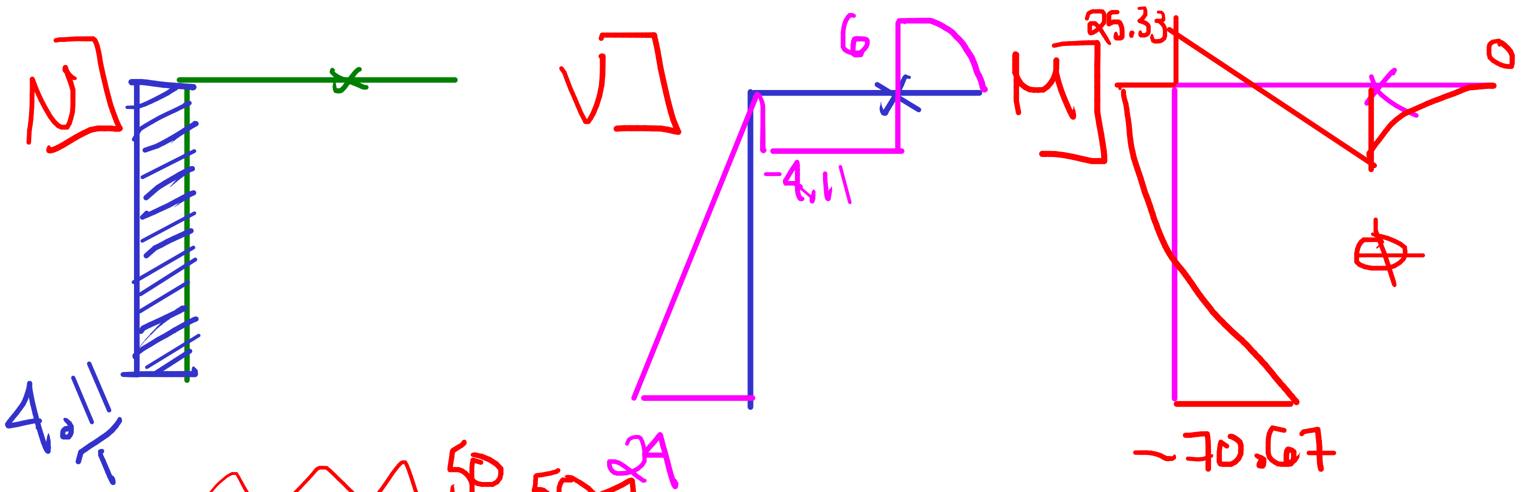
$$M_{BC} := \frac{2}{12} \cdot \left(2 \cdot \theta_B + \theta_C\right) = 25.3333$$

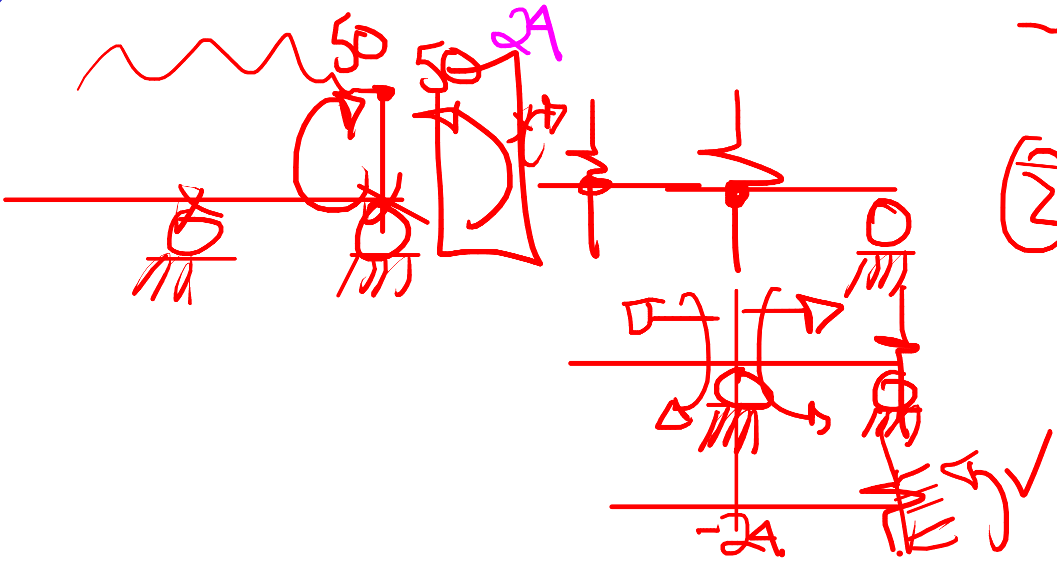
$$M_{CB} := \frac{2}{12} \cdot \left(\theta_B + 2 \cdot \theta_C\right) = 24$$



$$V_C = \frac{25.33 + 70.67}{8} = 12^*$$

$$V_V = \frac{25.33 + 24}{12} = 4.11$$

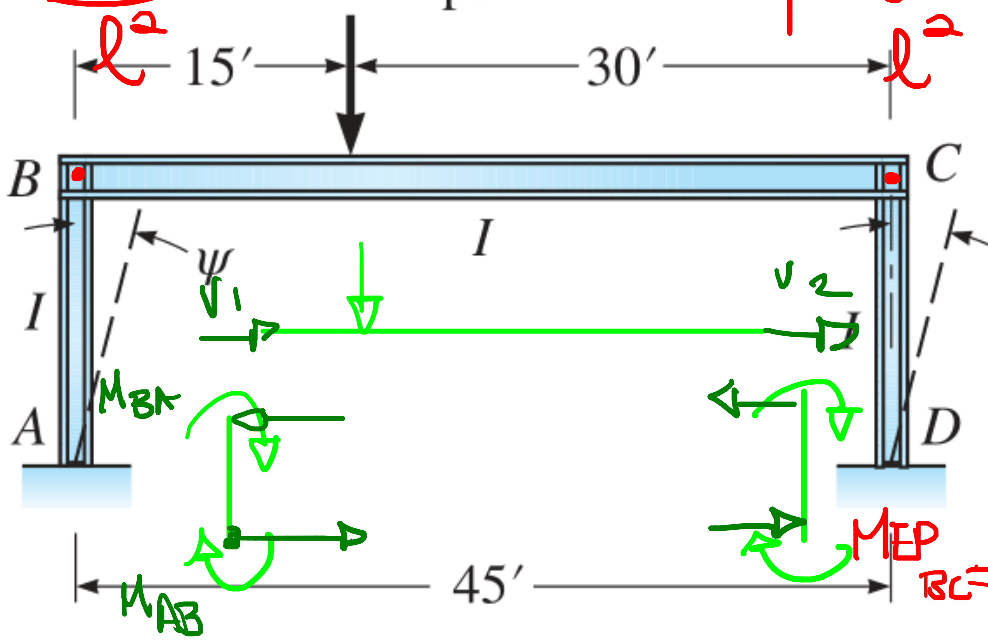




② $M = V$

$M = X$

$-\frac{Pb^2a}{l^2}$ $\qquad$ $+\frac{Pa^2b}{l^2}$



$I = 240 \text{ in}^4$

$E = 30{,}000 \text{ Ksi}$

$\Delta_B = ¿?$

$M^{EP}_{BC} = -\frac{12 \cdot 30^2 \cdot 15}{45^2} = -80$

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

$M^{EP}_{CB} = +\frac{12 \cdot 15^2 \cdot 30}{45^2} = 40$

$M_{AB} = \frac{2EI}{15}(\theta_B - 3\psi)$

$M_{BA} = \frac{2EI}{15}(2\theta_B - 3\psi)$

$M_{BC} = \frac{2EI}{15 \cdot 3}(2\theta_B + \theta_C) - 80$

$M_{CB} = \frac{2EI}{15 \cdot 3}(2\theta_C + \theta_B) + 40$

$$M_{CD} = \frac{2EI}{15}(2\theta_C - 3\psi)$$

$$K = \frac{EI}{15}$$

$$M_{DC} = \frac{2EI}{L}(\theta_C - 3\psi)$$

$$M_{AB} = 2K(\theta_B - 3\psi)$$

$$\Rightarrow M_{BA} = 2K(2\theta_B - 3\psi)$$

$$\Rightarrow M_{BC} = \frac{2}{3}K(2\theta_B + \theta_C) - 80$$

$$\Rightarrow M_{CB} = \frac{2}{3}K(2\theta_C + \theta_B) + 40$$

$$\Rightarrow M_{CD} = 2K(2\theta_C - 3\psi)$$

$$M_{DC} = 2K(\theta_C - 3\psi)$$

$$\frac{\cancel{2K\theta_B} - \cancel{6K\psi} + 4K\cancel{\theta_B} - \cancel{6K\psi}}{15} + \frac{\cancel{4K\theta_C} - \cancel{6K\psi} + \cancel{2K\theta_C} - \cancel{6K\psi}}{15} = 0$$

$$6K\theta_B + 6K\theta_C - 24K\psi = 0$$

$$K\theta_B + K\theta_C - 4K\psi = 0 \longrightarrow 3$$

$$\Sigma F_x = V_1 + V_2 = 0$$

$$\frac{M_{AB} + M_{BA}}{15} + \frac{M_{CD} + M_{DC}}{15} = 0$$

$$\Sigma M_B = 0 = M_{BA} + M_{BC}$$

$$= 4K\theta_B - 6K\psi + \frac{4}{3}K\theta_B + \frac{2}{3}K\theta_C - 80$$

$$0 = \frac{16}{3}K\theta_B + \frac{2}{3}K\theta_C - 6K\psi - 80$$

$$240 = 16K\theta_B + 2K\theta_C - 18K\psi$$

$$120 = 8K\theta_B + K\theta_C - 9K\psi \quad \text{———} \quad (1)$$

$$\Sigma M_C = M_{CB} + M_{CD} = 0$$

$$\frac{4}{3}K\theta_C + \frac{2}{3}K\theta_B + 40 + 4K\theta_C - 6K\psi = 0$$

$$\frac{2}{3}K\theta_B + \frac{16}{3}K\theta_C - 6K\psi + 40 = 0 \quad \sim -3$$

$$2K\theta_B + 16K\theta_C - 18K\psi = -120 \longleftarrow (2)$$

$$K\theta_B + 8K\theta_C - 9K\psi = -60$$

$$\begin{bmatrix} 8 & 1 & -9 \\ 2 & 16 & -18 \\ 1 & 1 & -4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 120 \\ -120 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{410}{21} \\ -\frac{130}{21} \\ \frac{10}{3} \end{bmatrix} \begin{matrix} \theta_B \\ \theta_C \\ \psi \end{matrix}$$

$$M_{AB} := 2\cdot\left(\theta_B - 3\cdot\psi\right) = 19.0476$$

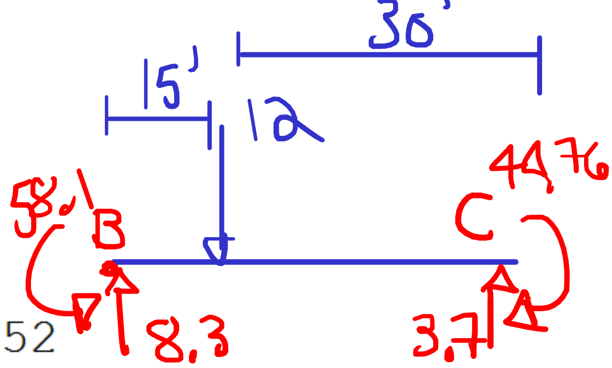
$$M_{BA} := 2\cdot\left(2\cdot\theta_B - 3\cdot\psi\right) = 58.0952$$

$$M_{BC} := \frac{2}{3}\cdot\left(2\cdot\theta_B + \theta_C\right) - 80 = -58.0952$$

$$M_{CB} := \frac{2}{3}\cdot\left(2\cdot\theta_C + \theta_B\right) + 40 = 44.7619$$

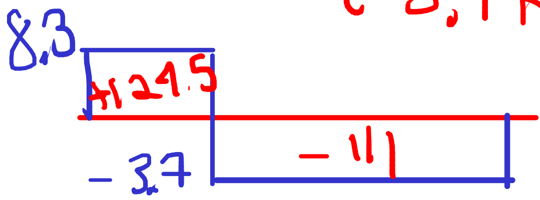
$$M_{CD} := 2\cdot\left(2\cdot\theta_C - 3\cdot\psi\right) = -44.7619$$

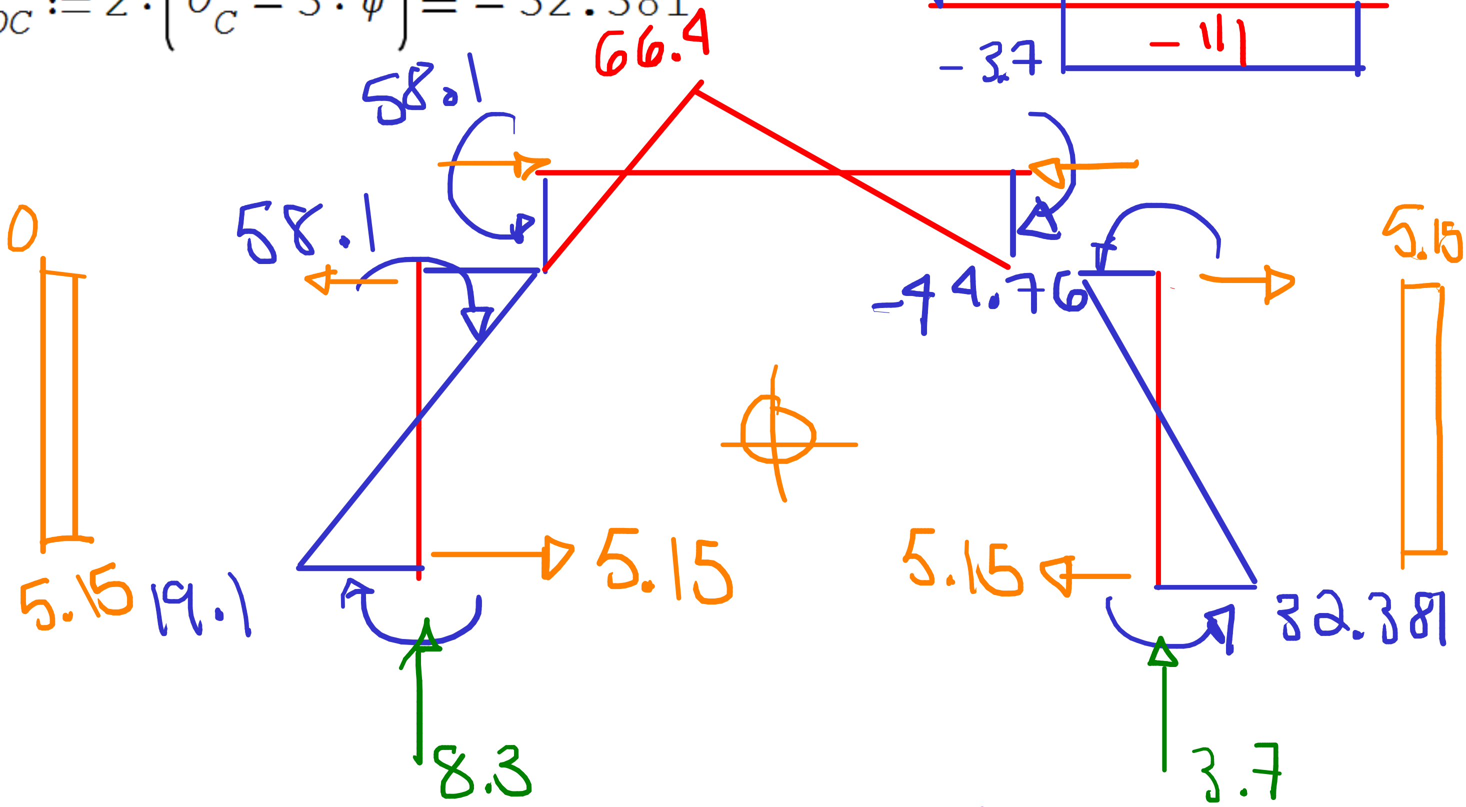
$$M_{DC} := 2\cdot\left(\theta_C - 3\cdot\psi\right) = -32.381$$

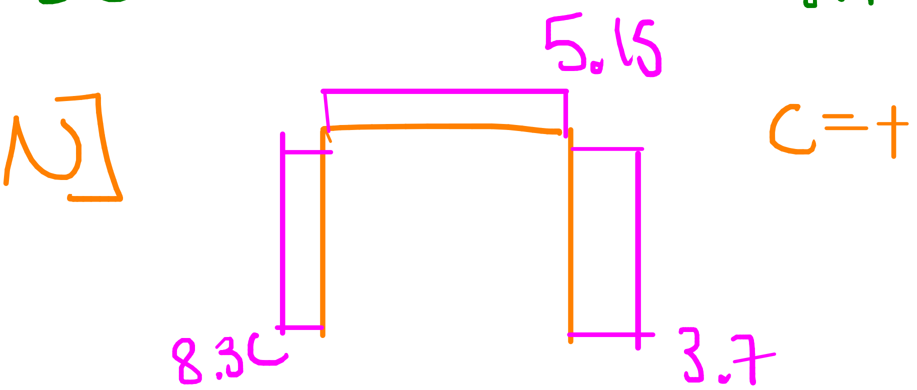


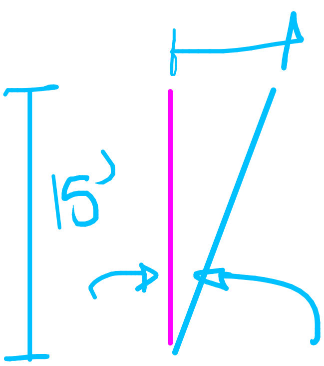
$$\Sigma M_B = -58.1 + 12(15) + 44.76 - V_C(45) = 0$$

$$V_C = 3.7\,K$$









$\psi = \dfrac{10}{3K} = \dfrac{10\cdot 15}{3EI}$ ← Usando todo en ft

$\psi = 0.000999$

$\Delta_B = 0.000999(15)(12) = 0.18\,in$

$$\psi = \frac{\Delta}{\ell}$$

$$\Delta = \psi \ell \checkmark$$

$$\tan(0.00999) = \frac{\Delta}{15}$$