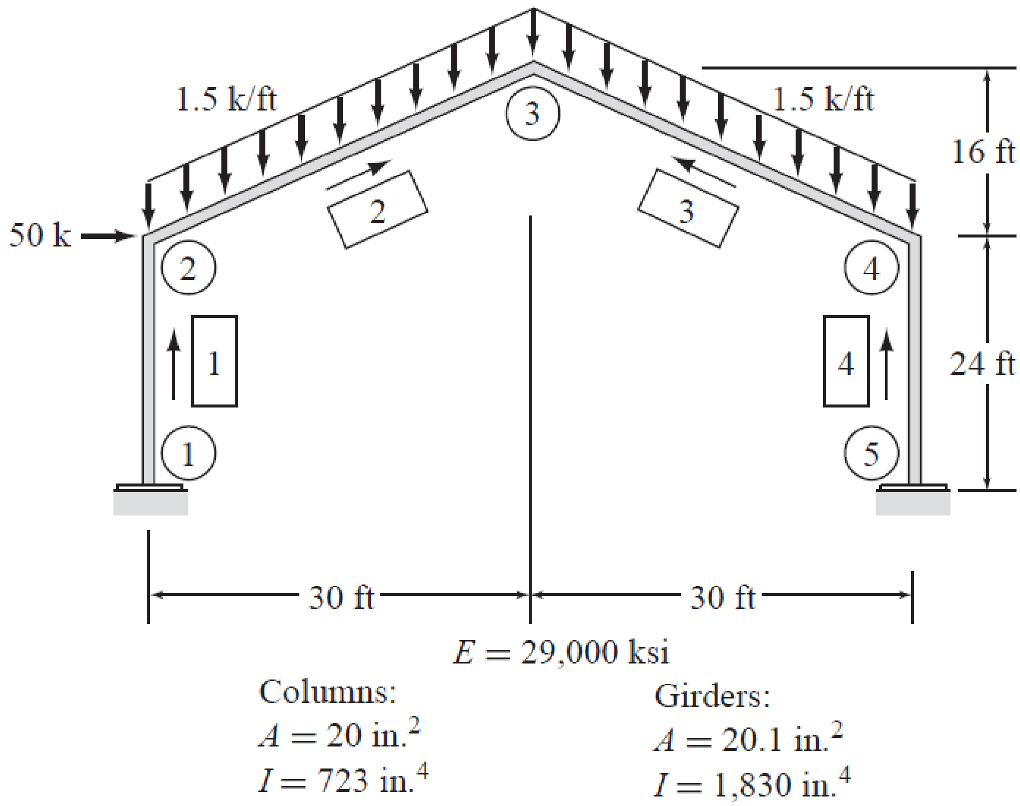
Ejemplo 6.49 Kassimali: Matrix Analysis of Structures.



Coordenadas:

$$x := \begin{bmatrix} 0 & 0 \\ 0 & 30 \\ 60 & 30 \\ 60 & 60 \end{bmatrix} \; ft$$

$$y := \begin{bmatrix} 0 & 24 \\ 24 & 40 \\ 24 & 40 \\ 0 & 24 \end{bmatrix} \; ft$$

Longitudes:

$$L := \begin{bmatrix} 24 \\ \sqrt{30^2 + 16^2} \\ \sqrt{30^2 + 16^2} \\ 24 \end{bmatrix} = \begin{bmatrix} 24 \\ 34 \\ 34 \\ 24 \end{bmatrix} \; ft$$

Información de entrada adicional. Unidades consistentes: Kip & Ft.

$$A := \begin{bmatrix} \frac{20}{12^2} \\ \frac{20.1}{12^2} \\ \frac{20.1}{12^2} \\ \frac{20}{12^2} \end{bmatrix} = \begin{bmatrix} 0.1389 \\ 0.1396 \\ 0.1396 \\ 0.1389 \end{bmatrix} \; ft^2 \qquad I := \begin{bmatrix} \frac{723}{12^4} \\ \frac{1830}{12^4} \\ \frac{1830}{12^4} \\ \frac{723}{12^4} \end{bmatrix} = \begin{bmatrix} 0.0349 \\ 0.0883 \\ 0.0883 \\ 0.0349 \end{bmatrix} \; ft^4 \qquad E := \begin{bmatrix} 29000 \cdot 12^2 \\ 29000 \cdot 12^2 \\ 29000 \cdot 12^2 \\ 29000 \cdot 12^2 \end{bmatrix} = \begin{bmatrix} 4.176 \cdot 10^6 \\ 4.176 \cdot 10^6 \\ 4.176 \cdot 10^6 \\ 4.176 \cdot 10^6 \end{bmatrix} \; ksf$$

Formación de las matrices de rigidez locales, k:

$$k(n) := \frac{E_n \cdot I_n}{L_n^{\,3}} \cdot \begin{bmatrix} \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & 12 & 6 \cdot L_n & 0 & -12 & 6 \cdot L_n \\ 0 & 6 \cdot L_n & 4 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 2 \cdot (L_n)^2 \\ -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & -12 & -6 \cdot L_n & 0 & 12 & -6 \cdot L_n \\ 0 & 6 \cdot L_n & 2 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 4 \cdot (L_n)^2 \end{bmatrix}$$

Matrices de transformación:

$$\text{seno}(n) := \frac{y_{n\,2} - y_{n\,1}}{L_n} \qquad \text{coseno}(n) := \frac{x_{n\,2} - x_{n\,1}}{L_n}$$

$$T(n) := \begin{bmatrix} \text{coseno}(n) & \text{seno}(n) & 0 & 0 & 0 & 0 \\ -\text{seno}(n) & \text{coseno}(n) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{coseno}(n) & \text{seno}(n) & 0 \\ 0 & 0 & 0 & -\text{seno}(n) & \text{coseno}(n) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrices de rigidez globales:

$$K(n) := T(n)^{T} \cdot k(n) \cdot T(n)$$

Implementación Numérica:

$$k_1 := k(1) = \begin{bmatrix} 24166.6667 & 0 & 0 & -24166.6667 & 0 & 0 \\ 0 & 126.3925 & 1516.7101 & 0 & -126.3925 & 1516.7101 \\ 0 & 1516.7101 & 24267.3611 & 0 & -1516.7101 & 12133.6806 \\ -24166.6667 & 0 & 0 & 24166.6667 & 0 & 0 \\ 0 & -126.3925 & -1516.7101 & 0 & 126.3925 & -1516.7101 \\ 0 & 1516.7101 & 12133.6806 & 0 & -1516.7101 & 24267.3611 \end{bmatrix}$$

$$T_1 := T(1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 11 & 12 & 1 & 2 & 3 \end{bmatrix}$$

$$K_1 := K(1) = \begin{bmatrix} 126.3925 & 0 & -1516.7101 & -126.3925 & 0 & -1516.7101 \\ 0 & 24166.6667 & 0 & 0 & -24166.6667 & 0 \\ -1516.7101 & 0 & 24267.3611 & 1516.7101 & 0 & 12133.6806 \\ -126.3925 & 0 & 1516.7101 & 126.3925 & 0 & 1516.7101 \\ 0 & -24166.6667 & 0 & 0 & 24166.6667 & 0 \\ -1516.7101 & 0 & 12133.6806 & 1516.7101 & 0 & 24267.3611 \end{bmatrix} \qquad \begin{bmatrix} 10 \\ 11 \\ 12 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$k_2 := k(2) = \begin{bmatrix} 17144.1176 & 0 & 0 & -17144.1176 & 0 & 0 \\ 0 & 112.5204 & 1912.846 & 0 & -112.5204 & 1912.846 \\ 0 & 1912.846 & 43357.8431 & 0 & -1912.846 & 21678.9216 \\ -17144.1176 & 0 & 0 & 17144.1176 & 0 & 0 \\ 0 & -112.5204 & -1912.846 & 0 & 112.5204 & -1912.846 \\ 0 & 1912.846 & 21678.9216 & 0 & -1912.846 & 43357.8431 \end{bmatrix}$$

$$T_2 := T(2) = \begin{bmatrix} 0.8824 & 0.4706 & 0 & 0 & 0 & 0 \\ -0.4706 & 0.8824 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8824 & 0.4706 & 0 \\ 0 & 0 & 0 & -0.4706 & 0.8824 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$K_2 := K(2) = \begin{bmatrix} 13372.4144 & 7071.9435 & -900.1628 & -13372.4144 & -7071.9435 & -900.1628 \\ 7071.9435 & 3884.2236 & 1687.8053 & -7071.9435 & -3884.2236 & 1687.8053 \\ -900.1628 & 1687.8053 & 43357.8431 & 900.1628 & -1687.8053 & 21678.9216 \\ -13372.4144 & -7071.9435 & 900.1628 & 13372.4144 & 7071.9435 & 900.1628 \\ -7071.9435 & -3884.2236 & -1687.8053 & 7071.9435 & 3884.2236 & -1687.8053 \\ -900.1628 & 1687.8053 & 21678.9216 & 900.1628 & -1687.8053 & 43357.8431 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$k_3 := k(3) = \begin{bmatrix} 17144.1176 & 0 & 0 & -17144.1176 & 0 & 0 \\ 0 & 112.5204 & 1912.846 & 0 & -112.5204 & 1912.846 \\ 0 & 1912.846 & 43357.8431 & 0 & -1912.846 & 21678.9216 \\ -17144.1176 & 0 & 0 & 17144.1176 & 0 & 0 \\ 0 & -112.5204 & -1912.846 & 0 & 112.5204 & -1912.846 \\ 0 & 1912.846 & 21678.9216 & 0 & -1912.846 & 43357.8431 \end{bmatrix}$$

$$T_3 := T(3) = \begin{bmatrix} -0.8824 & 0.4706 & 0 & 0 & 0 & 0 \\ -0.4706 & -0.8824 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.8824 & 0.4706 & 0 \\ 0 & 0 & 0 & -0.4706 & -0.8824 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8 & 9 & 4 & 5 & 6 \end{bmatrix}$$

$$K_3 := K(3) = \begin{bmatrix} 13372.4144 & -7071.9435 & -900.1628 & -13372.4144 & 7071.9435 & -900.1628 \\ -7071.9435 & 3884.2236 & -1687.8053 & 7071.9435 & -3884.2236 & -1687.8053 \\ -900.1628 & -1687.8053 & 43357.8431 & 900.1628 & 1687.8053 & 21678.9216 \\ -13372.4144 & 7071.9435 & 900.1628 & 13372.4144 & -7071.9435 & 900.1628 \\ 7071.9435 & -3884.2236 & 1687.8053 & -7071.9435 & 3884.2236 & 1687.8053 \\ -900.1628 & -1687.8053 & 21678.9216 & 900.1628 & 1687.8053 & 43357.8431 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 9 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$k_4 := k(4) = \begin{bmatrix} 24166.6667 & 0 & 0 & -24166.6667 & 0 & 0 \\ 0 & 126.3925 & 1516.7101 & 0 & -126.3925 & 1516.7101 \\ 0 & 1516.7101 & 24267.3611 & 0 & -1516.7101 & 12133.6806 \\ -24166.6667 & 0 & 0 & 24166.6667 & 0 & 0 \\ 0 & -126.3925 & -1516.7101 & 0 & 126.3925 & -1516.7101 \\ 0 & 1516.7101 & 12133.6806 & 0 & -1516.7101 & 24267.3611 \end{bmatrix}$$

$$T_4 := T(4) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 14 & 15 & 7 & 8 & 9 \end{bmatrix}$$

$$K_4 := K(4) = \begin{bmatrix} 126.3925 & 0 & -1516.7101 & -126.3925 & 0 & -1516.7101 \\ 0 & 24166.6667 & 0 & 0 & -24166.6667 & 0 \\ -1516.7101 & 0 & 24267.3611 & 1516.7101 & 0 & 12133.6806 \\ -126.3925 & 0 & 1516.7101 & 126.3925 & 0 & 1516.7101 \\ 0 & -24166.6667 & 0 & 0 & 24166.6667 & 0 \\ -1516.7101 & 0 & 12133.6806 & 1516.7101 & 0 & 24267.3611 \end{bmatrix} \quad \begin{bmatrix} 13 \\ 14 \\ 15 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Ensamblaje de la matriz de rigidez global del sistema:

$$S:=\begin{bmatrix}
K_{1\,44}+K_{2\,11} & K_{1\,45}+K_{2\,12} & K_{1\,46}+K_{2\,13} & K_{2\,14} & K_{2\,15} & K_{2\,16} & 0 & 0 & 0 \\
K_{1\,54}+K_{2\,21} & K_{1\,55}+K_{2\,22} & K_{1\,56}+K_{2\,23} & K_{2\,24} & K_{2\,25} & K_{2\,26} & 0 & 0 & 0 \\
K_{1\,64}+K_{2\,31} & K_{1\,65}+K_{2\,32} & K_{1\,66}+K_{2\,33} & K_{2\,34} & K_{2\,35} & K_{2\,36} & 0 & 0 & 0 \\
K_{2\,41} & K_{2\,42} & K_{2\,43} & K_{2\,44}+K_{3\,44} & K_{2\,45}+K_{3\,45} & K_{2\,46}+K_{3\,46} & K_{3\,41} & K_{3\,42} & K_{3\,43} \\
K_{2\,51} & K_{2\,52} & K_{2\,53} & K_{2\,54}+K_{3\,54} & K_{2\,55}+K_{3\,55} & K_{2\,56}+K_{3\,56} & K_{3\,51} & K_{3\,52} & K_{3\,53} \\
K_{2\,61} & K_{2\,62} & K_{2\,63} & K_{2\,64}+K_{3\,64} & K_{2\,65}+K_{3\,65} & K_{2\,66}+K_{3\,66} & K_{3\,61} & K_{3\,62} & K_{3\,63} \\
0 & 0 & 0 & K_{3\,14} & K_{3\,15} & K_{3\,16} & K_{3\,11}+K_{4\,44} & K_{3\,12}+K_{4\,45} & K_{3\,13}+K_{4} \\
0 & 0 & 0 & K_{3\,24} & K_{3\,25} & K_{3\,26} & K_{3\,21}+K_{4\,54} & K_{3\,22}+K_{4\,55} & K_{3\,23}+K_{4} \\
0 & 0 & 0 & K_{3\,34} & K_{3\,35} & K_{3\,36} & K_{3\,31}+K_{4\,64} & K_{3\,32}+K_{4\,65} & K_{3\,33}+K_{4}
\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

$$S=\begin{bmatrix}
13498.81 & 7071.94 & 616.55 & -13372.41 & -7071.94 & -900.16 & 0 & 0 & 0 \\
7071.94 & 28050.89 & 1687.81 & -7071.94 & -3884.22 & 1687.81 & 0 & 0 & 0 \\
616.55 & 1687.81 & 67625.2 & 900.16 & -1687.81 & 21678.92 & 0 & 0 & 0 \\
-13372.41 & -7071.94 & 900.16 & 26744.83 & 0 & 1800.33 & -13372.41 & 7071.94 & 900.16 \\
-7071.94 & -3884.22 & -1687.81 & 0 & 7768.45 & 0 & 7071.94 & -3884.22 & 1687.81 \\
-900.16 & 1687.81 & 21678.92 & 1800.33 & 0 & 86715.69 & -900.16 & -1687.81 & 21678.92 \\
0 & 0 & 0 & -13372.41 & 7071.94 & -900.16 & 13498.81 & -7071.94 & 616.55 \\
0 & 0 & 0 & 7071.94 & -3884.22 & -1687.81 & -7071.94 & 28050.89 & -1687.81 \\
0 & 0 & 0 & 900.16 & 1687.81 & 21678.92 & 616.55 & -1687.81 & 67625.2
\end{bmatrix}
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Formulación alternativa de la matriz de rigidez, usando submatrices:

$$DeK1:=\mathrm{submatrix}\left(K_1, 4, 6, 4, 6\right)=\begin{bmatrix}
126.3925 & 0 & 1516.7101 \\
0 & 24166.6667 & 0 \\
1516.7101 & 0 & 24267.3611
\end{bmatrix}$$

$$ExpDeK1:=\mathrm{augment}\left(\mathrm{stack}\left(DeK1, \mathrm{matrix}(6, 3)\right), \mathrm{matrix}(9, 6)\right)$$

$$ExpDeK1=\begin{bmatrix}
126.3925 & 0 & 1516.7101 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 24166.6667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1516.7101 & 0 & 24267.3611 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$DeK2:=K_2=\begin{bmatrix}
13372.4144 & 7071.9435 & -900.1628 & -13372.4144 & -7071.9435 & -900.1628 \\
7071.9435 & 3884.2236 & 1687.8053 & -7071.9435 & -3884.2236 & 1687.8053 \\
-900.1628 & 1687.8053 & 43357.8431 & 900.1628 & -1687.8053 & 21678.9216 \\
-13372.4144 & -7071.9435 & 900.1628 & 13372.4144 & 7071.9435 & 900.1628 \\
-7071.9435 & -3884.2236 & -1687.8053 & 7071.9435 & 3884.2236 & -1687.8053 \\
-900.1628 & 1687.8053 & 21678.9216 & 900.1628 & -1687.8053 & 43357.8431
\end{bmatrix}$$

$ExpDeK2 := augment(stack(DeK2, matrix(3, 6)), matrix(9, 3))$

$$ExpDeK2 = \begin{bmatrix} 13372.4144 & 7071.9435 & -900.1628 & -13372.4144 & -7071.9435 & -900.1628 & 0 & 0 & 0 \\ 7071.9435 & 3884.2236 & 1687.8053 & -7071.9435 & -3884.2236 & 1687.8053 & 0 & 0 & 0 \\ -900.1628 & 1687.8053 & 43357.8431 & 900.1628 & -1687.8053 & 21678.9216 & 0 & 0 & 0 \\ -13372.4144 & -7071.9435 & 900.1628 & 13372.4144 & 7071.9435 & 900.1628 & 0 & 0 & 0 \\ -7071.9435 & -3884.2236 & -1687.8053 & 7071.9435 & 3884.2236 & -1687.8053 & 0 & 0 & 0 \\ -900.1628 & 1687.8053 & 21678.9216 & 900.1628 & -1687.8053 & 43357.8431 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$DeK3_{ArribaIzquierda} := submatrix(K_3, 1, 3, 1, 3) = \begin{bmatrix} 13372.4144 & -7071.9435 & -900.1628 \\ -7071.9435 & 3884.2236 & -1687.8053 \\ -900.1628 & -1687.8053 & 43357.8431 \end{bmatrix}$$

$$DeK3_{AbajoIzquierda} := submatrix(K_3, 4, 6, 1, 3) = \begin{bmatrix} -13372.4144 & 7071.9435 & 900.1628 \\ 7071.9435 & -3884.2236 & 1687.8053 \\ -900.1628 & -1687.8053 & 21678.9216 \end{bmatrix}$$

$$DeK3_{ArribaDerecha} := submatrix(K_3, 1, 3, 4, 6) = \begin{bmatrix} -13372.4144 & 7071.9435 & -900.1628 \\ 7071.9435 & -3884.2236 & -1687.8053 \\ 900.1628 & 1687.8053 & 21678.9216 \end{bmatrix}$$

$$DeK3_{AbajoDerecha} := submatrix(K_3, 4, 6, 4, 6) = \begin{bmatrix} 13372.4144 & -7071.9435 & 900.1628 \\ -7071.9435 & 3884.2236 & 1687.8053 \\ 900.1628 & 1687.8053 & 43357.8431 \end{bmatrix}$$

$ExpDeK3_{ArrI} := augment(matrix(9, 6), stack(matrix(6, 3), DeK3_{ArribaIzquierda}))$

$$ExpDeK3_{ArrI} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 13372.4144 & -7071.9435 & -900.1628 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7071.9435 & 3884.2236 & -1687.8053 \\ 0 & 0 & 0 & 0 & 0 & 0 & -900.1628 & -1687.8053 & 43357.8431 \end{bmatrix}$$

$ExpDeK3_{AbD} := augment(matrix(9, 3), stack(matrix(3, 3), DeK3_{AbajoDerecha}, matrix(3, 3)), matrix(9, 3))$

$$ExpDeK3_{AbD} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13372.4144 & -7071.9435 & 900.1628 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7071.9435 & 3884.2236 & 1687.8053 & 0 & 0 & 0 \\ 0 & 0 & 0 & 900.1628 & 1687.8053 & 43357.8431 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$ExpDeK3_{AbI} := augment(matrix(9, 6), stack(matrix(3, 3), DeK3_{AbajoIzquierda}, matrix(3, 3)))$

$$ExpDeK3_{AbI}=\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-13372.4144&7071.9435&900.1628\\ 0&0&0&0&0&0&7071.9435&-3884.2236&1687.8053\\ 0&0&0&0&0&0&-900.1628&-1687.8053&21678.9216\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$ExpDeK3_{ArrD}:=\mathrm{augment}\left(\mathrm{matrix}(9,3),\ \mathrm{stack}\left(\mathrm{matrix}(6,3),\ DeK3_{ArribaDerecha}\right),\ \mathrm{matrix}(9,3)\right)$$

$$ExpDeK3_{ArrD}=\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&-13372.4144&7071.9435&-900.1628&0&0&0\\ 0&0&0&7071.9435&-3884.2236&-1687.8053&0&0&0\\ 0&0&0&900.1628&1687.8053&21678.9216&0&0&0 \end{bmatrix}$$

$$ExpDeK3:=ExpDeK3_{ArrI}+ExpDeK3_{ArrD}+ExpDeK3_{AbI}+ExpDeK3_{AbD}$$

$$ExpDeK3=\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&13372.4144&-7071.9435&900.1628&-13372.4144&7071.9435&900.1628\\ 0&0&0&-7071.9435&3884.2236&1687.8053&7071.9435&-3884.2236&1687.8053\\ 0&0&0&900.1628&1687.8053&43357.8431&-900.1628&-1687.8053&21678.9216\\ 0&0&0&-13372.4144&7071.9435&-900.1628&13372.4144&-7071.9435&-900.1628\\ 0&0&0&7071.9435&-3884.2236&-1687.8053&-7071.9435&3884.2236&-1687.8053\\ 0&0&0&900.1628&1687.8053&21678.9216&-900.1628&-1687.8053&43357.8431 \end{bmatrix}$$

$$DeK4:=\mathrm{submatrix}\left(K_4,\ 4,\ 6,\ 4,\ 6\right)=\begin{bmatrix} 126.3925&0&1516.7101\\ 0&24166.6667&0\\ 1516.7101&0&24267.3611 \end{bmatrix}$$

$$ExpDeK4:=\mathrm{augment}\left(\mathrm{matrix}(9,6),\ \mathrm{stack}\left(\mathrm{matrix}(6,3),\ DeK4\right)\right)$$

$$ExpDeK4=\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&126.3925&0&1516.7101\\ 0&0&0&0&0&0&0&24166.6667&0\\ 0&0&0&0&0&0&1516.7101&0&24267.3611 \end{bmatrix}$$

SS:= ExpDeK1+ ExpDeK2+ ExpDeK3+ ExpDeK4

$$SS=\begin{bmatrix} 13498.8069 & 7071.9435 & 616.5472 & -13372.4144 & -7071.9435 & -900.1628 & 0 & 0 & 0 \\ 7071.9435 & 28050.8902 & 1687.8053 & -7071.9435 & -3884.2236 & 1687.8053 & 0 & 0 & 0 \\ 616.5472 & 1687.8053 & 67625.2042 & 900.1628 & -1687.8053 & 21678.9216 & 0 & 0 & 0 \\ -13372.4144 & -7071.9435 & 900.1628 & 26744.8289 & 0 & 1800.3257 & -13372.4144 & 7071.9435 & 900.1628 \\ -7071.9435 & -3884.2236 & -1687.8053 & 0 & 7768.4471 & 0 & 7071.9435 & -3884.2236 & 1687.8053 \\ -900.1628 & 1687.8053 & 21678.9216 & 1800.3257 & 0 & 86715.6863 & -900.1628 & -1687.8053 & 21678.9216 \\ 0 & 0 & 0 & -13372.4144 & 7071.9435 & -900.1628 & 13498.8069 & -7071.9435 & 616.5472 \\ 0 & 0 & 0 & 7071.9435 & -3884.2236 & -1687.8053 & -7071.9435 & 28050.8902 & -1687.8053 \\ 0 & 0 & 0 & 900.1628 & 1687.8053 & 21678.9216 & 616.5472 & -1687.8053 & 67625.2042 \end{bmatrix}$$

$$SS-S=\begin{bmatrix} 5.4\cdot10^{-11} & 1.6\cdot10^{-11} & 0 & 2\cdot10^{-11} & -1.6\cdot10^{-11} & 0 & 0 & 0 & 0 \\ 1.6\cdot10^{-11} & 4.3\cdot10^{-10} & 0 & -1.6\cdot10^{-11} & -1.8\cdot10^{-11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\cdot10^{-11} & -1.6\cdot10^{-11} & 0 & -6.8\cdot10^{-11} & 0 & 0 & 2\cdot10^{-11} & 1.6\cdot10^{-11} & 0 \\ -1.6\cdot10^{-11} & -1.8\cdot10^{-11} & 0 & 0 & 9.3\cdot10^{-12} & 0 & 1.6\cdot10^{-11} & -1.8\cdot10^{-11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8.7\cdot10^{-12} & 1.6\cdot10^{-11} & 0 & 4.8\cdot10^{-11} & -1.6\cdot10^{-11} & 0 \\ 0 & 0 & 0 & 1.6\cdot10^{-11} & -2.4\cdot10^{-11} & 0 & -1.6\cdot10^{-11} & 1.6\cdot10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Como puede apreciar, tanto el método "manual" como el de usar submatrices funcionan de la misma manera. Augment añade elementos horizontalmente, y Stack, verticalmente. La función Matrix hace matrices en ceros. Sume matrices del mismo orden.

Cargas

$$N:=\begin{bmatrix} 0 & 0 \\ 1.5\cdot\frac{16}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} & 1.5\cdot\frac{16}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} \\ 1.5\cdot\frac{16}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} & 1.5\cdot\frac{16}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 12 & 12 \\ 12 & 12 \\ 0 & 0 \end{bmatrix} \quad kip$$

$$V:=\begin{bmatrix} 0 & 0 \\ 1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} & 1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} \\ -1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} & -1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\sqrt{16^2+30^2}}{2} \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 22.5 & 22.5 \\ -22.5 & -22.5 \\ 0 & 0 \end{bmatrix} \quad kip$$

$$M:=\begin{bmatrix} 0 & 0 \\ 1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\left(\sqrt{16^2+30^2}\right)^2}{12} & -1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\left(\sqrt{16^2+30^2}\right)^2}{12} \\ -1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\left(\sqrt{16^2+30^2}\right)^2}{12} & 1.5\cdot\frac{30}{\sqrt{16^2+30^2}}\cdot\frac{\left(\sqrt{16^2+30^2}\right)^2}{12} \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 127.5 & -127.5 \\ -127.5 & 127.5 \\ 0 & 0 \end{bmatrix} \quad kip\ ft$$

Fabricación del vector de fuerzas locales, Q.f.

$$Q_f(n) := \begin{bmatrix} N_{n\,1} \\ V_{n\,1} \\ M_{n\,1} \\ N_{n\,2} \\ V_{n\,2} \\ M_{n\,2} \end{bmatrix}$$

$$Q_{f1} := Q_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad Q_{f2} := Q_f(2) = \begin{bmatrix} 12 \\ 22.5 \\ 127.5 \\ 12 \\ 22.5 \\ -127.5 \end{bmatrix} \quad Q_{f3} := Q_f(3) = \begin{bmatrix} 12 \\ -22.5 \\ -127.5 \\ 12 \\ -22.5 \\ 127.5 \end{bmatrix} \quad Q_{f4} := Q_f(4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Transformación de los vectores de fuerzas locales a fuerzas globales (F.f)

$$F_f(n) := T(n)^{-1} \cdot Q_f(n)$$

$$F_{f1} := F_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad F_{f2} := F_f(2) = \begin{bmatrix} 0 \\ 25.5 \\ 127.5 \\ 0 \\ 25.5 \\ -127.5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad F_{f3} := F_f(3) = \begin{bmatrix} 0 \\ 25.5 \\ -127.5 \\ 0 \\ 25.5 \\ 127.5 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad F_{f4} := F_f(4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 13 \\ 14 \\ 15 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Vector de fuerzas nodales P:

$$P := \begin{bmatrix} 50 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Solución del sistema de ecuaciones:

$$d := S^{-1} \cdot \left(P - P_f\right) = \begin{bmatrix} 0.2415 \\ -0.0018 \\ -0.0117 \\ 0.2917 \\ -0.101 \\ 0.0039 \\ 0.341 \\ -0.0025 \\ -0.0039 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Vector de fuerzas equivalentes P.f

$$P_f := \begin{bmatrix} F_{f1_4} + F_{f2_1} \\ F_{f1_5} + F_{f2_2} \\ F_{f1_6} + F_{f2_3} \\ F_{f2_4} + F_{f3_4} \\ F_{f2_5} + F_{f3_5} \\ F_{f2_6} + F_{f3_6} \\ F_{f3_1} + F_{f4_4} \\ F_{f3_2} + F_{f4_5} \\ F_{f3_3} + F_{f4_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 25.5 \\ 127.5 \\ 0 \\ 51 \\ 0 \\ 0 \\ 25.5 \\ -127.5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Vectores de desplazamientos en coordenadas globales v, y locales u.

$$v_1 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2415 \\ -0.0018 \\ -0.0117 \end{bmatrix} \quad \begin{bmatrix} 10 \\ 11 \\ 12 \\ 1 \\ 2 \\ 3 \end{bmatrix} \qquad u_1 := T(1) \cdot v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0018 \\ -0.2415 \\ -0.0117 \end{bmatrix}$$

$$v_2 := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0.2415 \\ -0.0018 \\ -0.0117 \\ 0.2917 \\ -0.101 \\ 0.0039 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \qquad u_2 := T(2) \cdot v_2 = \begin{bmatrix} 0.2122 \\ -0.1152 \\ -0.0117 \\ 0.2099 \\ -0.2264 \\ 0.0039 \end{bmatrix}$$

$$v_3 := \begin{bmatrix} d_7 \\ d_8 \\ d_9 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0.341 \\ -0.0025 \\ -0.0039 \\ 0.2917 \\ -0.101 \\ 0.0039 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 9 \\ 4 \\ 5 \\ 6 \end{bmatrix} \qquad u_3 := T(3) \cdot v_3 = \begin{bmatrix} -0.3021 \\ -0.1583 \\ -0.0039 \\ -0.3049 \\ -0.0482 \\ 0.0039 \end{bmatrix}$$

$$v_4 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_7 \\ d_8 \\ d_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.341 \\ -0.0025 \\ -0.0039 \end{bmatrix} \quad \begin{bmatrix} 13 \\ 14 \\ 15 \\ 7 \\ 8 \\ 9 \end{bmatrix} \qquad u_4 := T(4) \cdot v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0025 \\ -0.341 \\ -0.0039 \end{bmatrix}$$

Fuerzas finales en el SCL (Q) y en el SCG (F):

$$Q_1 := k_1 \cdot u_1 + Q_{f1} = \begin{bmatrix} 42.5749 \\ 12.7961 \\ 224.4562 \\ -42.5749 \\ -12.7961 \\ 82.6492 \end{bmatrix} \qquad Q_2 := k_2 \cdot u_2 + Q_{f2} = \begin{bmatrix} 52.8623 \\ 20.0584 \\ -82.6492 \\ -28.8623 \\ 24.9416 \\ -0.3663 \end{bmatrix}$$

$$Q_3 := k_3 \cdot u_3 + Q_{f3} = \begin{bmatrix} 60.7918 \\ -34.9262 \\ -422.8555 \\ -36.7918 \\ -10.0738 \\ 0.3663 \end{bmatrix} \qquad Q_4 := k_4 \cdot u_4 + Q_{f4} = \begin{bmatrix} 59.4251 \\ 37.2039 \\ 470.0391 \\ -59.4251 \\ -37.2039 \\ 422.8555 \end{bmatrix}$$

$$F_1 := K_1 \cdot v_1 + F_{f1} = \begin{bmatrix} -12.7961 \\ 42.5749 \\ 224.4562 \\ 12.7961 \\ -42.5749 \\ 82.6492 \end{bmatrix} \quad \begin{bmatrix} 10 \\ 11 \\ 12 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$F_2 := K_2 \cdot v_2 + F_{f2} = \begin{bmatrix} 37.2039 \\ 42.5749 \\ -82.6492 \\ -37.2039 \\ 8.4251 \\ -0.3663 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$F_3 := K_3 \cdot v_3 + F_{f3} = \begin{bmatrix} -37.2039 \\ 59.4251 \\ -422.8555 \\ 37.2039 \\ -8.4251 \\ 0.3663 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 9 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

$$F_4 := K_4 \cdot v_4 + F_{f4} = \begin{bmatrix} -37.2039 \\ 59.4251 \\ 470.0391 \\ 37.2039 \\ -59.4251 \\ 422.8555 \end{bmatrix} \quad \begin{bmatrix} 13 \\ 14 \\ 15 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

$$Reacciones = \begin{bmatrix} F_{1_1} \\ F_{1_2} \\ F_{1_3} \\ F_{4_1} \\ F_{4_2} \\ F_{4_3} \end{bmatrix} = \begin{bmatrix} -12.7961 \\ 42.5749 \\ 224.4562 \\ -37.2039 \\ 59.4251 \\ 470.0391 \end{bmatrix} \quad \begin{bmatrix} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{bmatrix} \quad \begin{bmatrix} kip \\ kip \\ kip\ ft \\ kip \\ kip \\ kip\ ft \end{bmatrix}$$