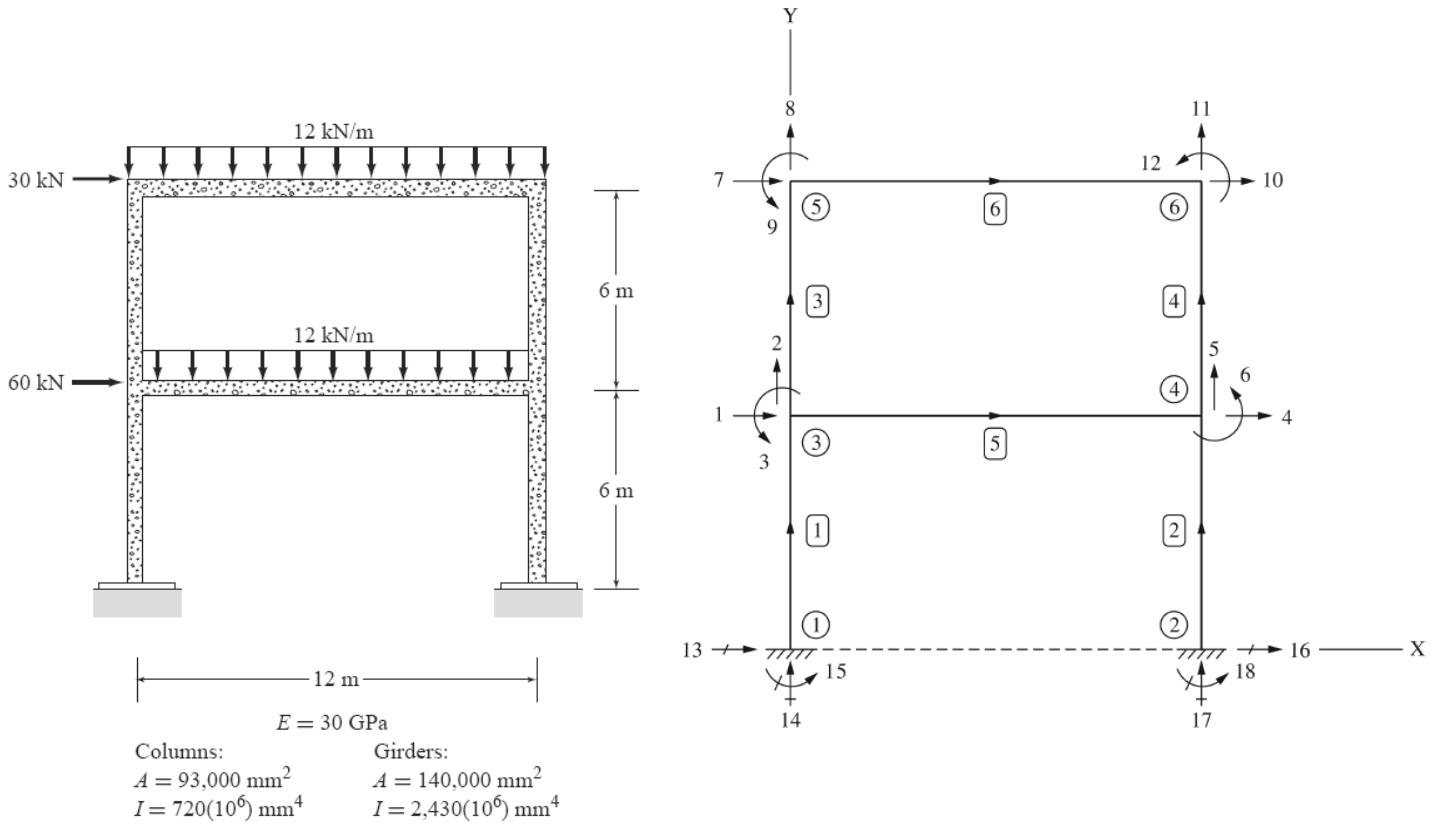


## Ejemplo de Marco Hiperestático, con el Método de las Rigideces



### Información de entrada

$$A := \begin{bmatrix} \frac{93000}{1000^2} \\ \frac{93000}{1000^2} \\ \frac{140000}{1000^2} \\ \frac{140000}{1000^2} \\ \frac{93000}{1000^2} \\ \frac{93000}{1000^2} \end{bmatrix} \text{ m}^2$$

$$I := \begin{bmatrix} \frac{720 \cdot 10^6}{1000^4} \\ \frac{720 \cdot 10^6}{1000^4} \\ \frac{2430 \cdot 10^6}{1000^4} \\ \frac{2430 \cdot 10^6}{1000^4} \\ \frac{720 \cdot 10^6}{1000^4} \\ \frac{720 \cdot 10^6}{1000^4} \end{bmatrix} \text{ m}^4$$

$$L := \begin{bmatrix} 6 \\ 6 \\ 6 \\ 6 \\ 12 \\ 12 \end{bmatrix} \text{ m}$$

$$E := \begin{bmatrix} 30 \cdot 10^6 \\ 30 \cdot 10^6 \\ 30 \cdot 10^6 \\ 30 \cdot 10^6 \\ 30 \cdot 10^6 \\ 30 \cdot 10^6 \end{bmatrix} \text{ kPa}$$

$$x := \begin{bmatrix} 0 & 0 \\ 12 & 12 \\ 0 & 0 \\ 12 & 12 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$Y := \begin{bmatrix} 0 & 6 \\ 0 & 6 \\ 6 & 12 \\ 6 & 12 \\ 6 & 6 \\ 12 & 12 \end{bmatrix}$$

$$V := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{12 \cdot 12}{2} & \frac{12 \cdot 12}{2} \\ \frac{12 \cdot 12}{2} & \frac{12 \cdot 12}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 72 & 72 \\ 72 & 72 \end{bmatrix}$$

$$N := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{12 \cdot 12^2}{12} & -\frac{12 \cdot 12^2}{12} \\ \frac{12 \cdot 12^2}{12} & -\frac{12 \cdot 12^2}{12} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 144 & -144 \\ 144 & -144 \end{bmatrix}$$

☐ - Fórmulas

$$k(n) := \frac{E_n \cdot I_n}{L_n^3} \cdot \begin{bmatrix} \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & 12 & 6 \cdot L_n & 0 & -12 & 6 \cdot L_n \\ 0 & 6 \cdot L_n & 4 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 2 \cdot (L_n)^2 \\ -\frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 & \frac{A_n \cdot (L_n)^2}{I_n} & 0 & 0 \\ 0 & -12 & -6 \cdot L_n & 0 & 12 & -6 \cdot L_n \\ 0 & 6 \cdot L_n & 2 \cdot (L_n)^2 & 0 & -6 \cdot L_n & 4 \cdot (L_n)^2 \end{bmatrix}$$

$$\text{seno}(n) := \frac{y_{n2} - y_{n1}}{L_n} \quad \text{coseno}(n) := \frac{x_{n2} - x_{n1}}{L_n}$$

$$T(n) := \begin{bmatrix} \text{coseno}(n) & \text{seno}(n) & 0 & 0 & 0 & 0 \\ -\text{seno}(n) & \text{coseno}(n) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{coseno}(n) & \text{seno}(n) & 0 \\ 0 & 0 & 0 & -\text{seno}(n) & \text{coseno}(n) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K(n) := T(n)^T \cdot k(n) \cdot T(n)$$

$$Q_f(n) := \begin{bmatrix} N_{n1} \\ V_{n1} \\ M_{n1} \\ N_{n2} \\ V_{n2} \\ M_{n2} \end{bmatrix}$$

$$F_f(n) := T(n)^{-1} \cdot Q_f(n)$$

Matrices de rigidez de las barras

Barras 1, 2, 3, 4

$$k_1 := k(1) = \begin{bmatrix} 4.65 \cdot 10^5 & 0 & 0 & -4.65 \cdot 10^5 & 0 & 0 \\ 0 & 1200 & 3600 & 0 & -1200 & 3600 \\ 0 & 3600 & 14400 & 0 & -3600 & 7200 \\ -4.65 \cdot 10^5 & 0 & 0 & 4.65 \cdot 10^5 & 0 & 0 \\ 0 & -1200 & -3600 & 0 & 1200 & -3600 \\ 0 & 3600 & 7200 & 0 & -3600 & 14400 \end{bmatrix} \quad \begin{array}{l} k_2 := k(2) \quad T_2 := T(2) \quad K_2 := K(2) \\ k_3 := k(3) \quad T_3 := T(3) \quad K_3 := K(3) \\ k_4 := k(4) \quad T_4 := T(4) \quad K_4 := K(4) \end{array}$$

$$T_1 := T(1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 6 & 10 & 11 & 12 \\ 1 & 2 & 3 & 7 & 8 & 9 \\ 0 & 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$K_1 := K(1) = \begin{bmatrix} 1200 & 0 & -3600 & -1200 & 0 & -3600 \\ 0 & 4.65 \cdot 10^5 & 0 & 0 & -4.65 \cdot 10^5 & 0 \\ -3600 & 0 & 14400 & 3600 & 0 & 7200 \\ -1200 & 0 & 3600 & 1200 & 0 & 3600 \\ 0 & -4.65 \cdot 10^5 & 0 & 0 & 4.65 \cdot 10^5 & 0 \\ -3600 & 0 & 7200 & 3600 & 0 & 14400 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 10 \\ 11 \\ 12 \end{bmatrix}$$

Barras 5 y 6

$$k_5 := k(5) = \begin{bmatrix} 3.5 \cdot 10^5 & 0 & 0 & -3.5 \cdot 10^5 & 0 & 0 \\ 0 & 506.25 & 3037.5 & 0 & -506.25 & 3037.5 \\ 0 & 3037.5 & 24300 & 0 & -3037.5 & 12150 \\ -3.5 \cdot 10^5 & 0 & 0 & 3.5 \cdot 10^5 & 0 & 0 \\ 0 & -506.25 & -3037.5 & 0 & 506.25 & -3037.5 \\ 0 & 3037.5 & 12150 & 0 & -3037.5 & 24300 \end{bmatrix}$$

$$T_5 := T(5) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$k_6 := k(6) \quad T_6 := T(6) \quad K_6 := K(6)$$

$$\begin{bmatrix} 7 & 8 & 9 & 10 & 11 & 12 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$K_5 := K(5) = \begin{bmatrix} 3.5 \cdot 10^5 & 0 & 0 & -3.5 \cdot 10^5 & 0 & 0 \\ 0 & 506.25 & 3037.5 & 0 & -506.25 & 3037.5 \\ 0 & 3037.5 & 24300 & 0 & -3037.5 & 12150 \\ -3.5 \cdot 10^5 & 0 & 0 & 3.5 \cdot 10^5 & 0 & 0 \\ 0 & -506.25 & -3037.5 & 0 & 506.25 & -3037.5 \\ 0 & 3037.5 & 12150 & 0 & -3037.5 & 24300 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$$

$$S := \begin{bmatrix} K_{1 4 4} + K_{3 1 1} + K_{5 1 1} & K_{1 4 5} + K_{3 1 2} + K_{5 1 2} & K_{1 4 6} + K_{3 1 3} + K_{5 1 3} & K_{5 1 4} & K_{5 1 5} \\ K_{1 5 4} + K_{3 2 1} + K_{5 2 1} & K_{1 5 5} + K_{3 2 2} + K_{5 2 2} & K_{1 5 6} + K_{3 2 3} + K_{5 2 3} & K_{5 2 4} & K_{5 2 5} \\ K_{1 6 4} + K_{3 3 1} + K_{5 3 1} & K_{1 6 5} + K_{3 3 2} + K_{5 3 2} & K_{1 6 6} + K_{3 3 3} + K_{5 3 3} & K_{5 3 4} & K_{5 3 5} \\ K_{5 4 1} & K_{5 4 2} & K_{5 4 3} & K_{2 4 4} + K_{4 1 1} + K_{5 4 4} & K_{2 4 5} + K_{4 1 2} + K_{5 4 5} & K_2 \\ K_{5 5 1} & K_{5 5 2} & K_{5 5 3} & K_{2 5 4} + K_{4 2 1} + K_{5 5 4} & K_{2 5 5} + K_{4 2 2} + K_{5 5 5} & K_2 \\ K_{5 6 1} & K_{5 6 2} & K_{5 6 3} & K_{2 6 4} + K_{4 3 1} + K_{5 6 4} & K_{2 6 5} + K_{4 3 2} + K_{5 6 5} & K_2 \\ K_{3 4 1} & K_{3 4 2} & K_{3 4 3} & 0 & 0 \\ K_{3 5 1} & K_{3 5 2} & K_{3 5 3} & 0 & 0 \\ K_{3 6 1} & K_{3 6 2} & K_{3 6 3} & 0 & 0 \\ 0 & 0 & 0 & K_{4 4 1} & K_{4 4 2} \\ 0 & 0 & 0 & K_{4 5 1} & K_{4 5 2} \\ 0 & 0 & 0 & K_{4 6 1} & K_{4 6 2} \end{bmatrix}$$

$$S = \begin{bmatrix} 3.524 \cdot 10^5 & 0 & 0 & -3.5 \cdot 10^5 & 0 & 0 & -1200 & 0 & -3600 & 0 & 0 \\ 0 & 9.3051 \cdot 10^5 & 3037.5 & 0 & -506.25 & 3037.5 & 0 & -4.65 \cdot 10^5 & 0 & 0 & 0 \\ 0 & 3037.5 & 53100 & 0 & -3037.5 & 12150 & 3600 & 0 & 7200 & 0 & 0 \\ -3.5 \cdot 10^5 & 0 & 0 & 3.524 \cdot 10^5 & 0 & 0 & 0 & 0 & 0 & -1200 & 0 \\ 0 & -506.25 & -3037.5 & 0 & 9.3051 \cdot 10^5 & -3037.5 & 0 & 0 & 0 & 0 & -4.65 \cdot 10^5 \\ 0 & 3037.5 & 12150 & 0 & -3037.5 & 53100 & 0 & 0 & 0 & 3600 & 0 \\ -1200 & 0 & 3600 & 0 & 0 & 0 & 3.512 \cdot 10^5 & 0 & 3600 & -3.5 \cdot 10^5 & 0 \\ 0 & -4.65 \cdot 10^5 & 0 & 0 & 0 & 0 & 0 & 4.6551 \cdot 10^5 & 3037.5 & 0 & -506.25 \\ -3600 & 0 & 7200 & 0 & 0 & 0 & 3600 & 3037.5 & 38700 & 0 & -3037.5 \\ 0 & 0 & 0 & -1200 & 0 & 3600 & -3.5 \cdot 10^5 & 0 & 0 & 3.512 \cdot 10^5 & 0 \\ 0 & 0 & 0 & 0 & -4.65 \cdot 10^5 & 0 & 0 & -506.25 & -3037.5 & 0 & 4.6551 \cdot 10^5 \\ 0 & 0 & 0 & -3600 & 0 & 7200 & 0 & 3037.5 & 12150 & 3600 & -3037.5 \end{bmatrix}$$

Vectores de fuerzas:

$$Q_{f1} := Q_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad F_{f1} := F_f(1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 5 \\ 6 \\ 10 \\ 11 \\ 12 \end{bmatrix} \quad \begin{matrix} Q_{f2} := Q_f(2) & F_{f2} := F_f(2) \\ Q_{f3} := Q_f(3) & F_{f3} := F_f(3) \\ Q_{f4} := Q_f(4) & F_{f4} := F_f(4) \end{matrix}$$

$$Q_{f5} := Q_f(5) = \begin{bmatrix} 0 \\ 72 \\ 144 \\ 0 \\ 72 \\ -144 \end{bmatrix} \quad F_{f5} := F_f(5) = \begin{bmatrix} 0 \\ 72 \\ 144 \\ 0 \\ 72 \\ -144 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} \quad \begin{matrix} Q_{f6} := Q_f(6) & F_{f6} := F_f(6) \end{matrix}$$

Vector de fuerzas de nodo fijo de la estructura

Vector de cargas nodales

$$P := \begin{bmatrix} 60 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 30 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$$

$$P_f := \begin{bmatrix} F_{f5_1} \\ F_{f5_2} \\ F_{f5_3} \\ F_{f5_4} \\ F_{f5_5} \\ F_{f5_6} \\ F_{f6_1} \\ F_{f6_2} \\ F_{f6_3} \\ F_{f6_4} \\ F_{f6_5} \\ F_{f6_6} \end{bmatrix} = \begin{bmatrix} 0 \\ 72 \\ 144 \\ 0 \\ 72 \\ -144 \\ 0 \\ 72 \\ 144 \\ 0 \\ 72 \\ -144 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$$

Solución del sistema

$$d := S^{-1} \cdot (P - P_f) = \begin{bmatrix} 0.0503 \\ -0.0002 \\ -0.0007 \\ 0.0503 \\ -0.0004 \\ -0.0016 \\ 0.0802 \\ -0.0004 \\ -0.0062 \\ 0.0801 \\ -0.0006 \\ 0.0032 \end{bmatrix}$$

Fuerzas en los extremos de los miembros y sus desplazamientos

$$v_1 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0503 \\ -0.0002 \\ -0.0007 \end{bmatrix}$$

$$u_1 := T(1) \cdot v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0002 \\ -0.0503 \\ -0.0007 \end{bmatrix}$$

$$Q_1 := k_1 \cdot u_1 + Q_{f1} = \begin{bmatrix} 109.0558 \\ 35.3182 \\ 130.9934 \\ -109.0558 \\ -35.3182 \\ 80.9156 \end{bmatrix}$$

$$F_{f1} := K_1 \cdot v_1 + F_{f1} = \begin{bmatrix} -35.3182 \\ 109.0558 \\ 130.9934 \\ 35.3182 \\ -109.0558 \\ 80.9156 \end{bmatrix}$$

$$v_2 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0503 \\ -0.0004 \\ -0.0016 \end{bmatrix}$$

$$u_2 := T(2) \cdot v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0004 \\ -0.0503 \\ -0.0016 \end{bmatrix}$$

$$Q_2 := k_2 \cdot u_2 + Q_{f2} = \begin{bmatrix} 178.9442 \\ 54.6818 \\ 169.6758 \\ -178.9442 \\ -54.6818 \\ 158.4153 \end{bmatrix}$$

$$F_{f2} := K_2 \cdot v_2 + F_{f2} = \begin{bmatrix} -54.6818 \\ 178.9442 \\ 169.6758 \\ 54.6818 \\ -178.9442 \\ 158.4153 \end{bmatrix}$$

$$v_3 := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_7 \\ d_8 \\ d_9 \end{bmatrix} = \begin{bmatrix} 0.0503 \\ -0.0002 \\ -0.0007 \\ 0.0802 \\ -0.0004 \\ -0.0062 \end{bmatrix}$$

$$u_3 := T(3) \cdot v_3 = \begin{bmatrix} -0.0002 \\ -0.0503 \\ -0.0007 \\ -0.0004 \\ -0.0802 \\ -0.0062 \end{bmatrix}$$

$$Q_3 := k_3 \cdot u_3 + Q_{f3} = \begin{bmatrix} 62.8568 \\ -11.5691 \\ -37.3576 \\ -62.8568 \\ 11.5691 \\ -32.0572 \end{bmatrix}$$

$$F_{f3} := K_3 \cdot v_3 + F_{f3} = \begin{bmatrix} 11.5691 \\ 62.8568 \\ -37.3576 \\ -11.5691 \\ -62.8568 \\ -32.0572 \end{bmatrix}$$

$$v_4 := \begin{bmatrix} d_4 \\ d_5 \\ d_6 \\ d_{10} \\ d_{11} \\ d_{12} \end{bmatrix} = \begin{bmatrix} 0.0503 \\ -0.0004 \\ -0.0016 \\ 0.0801 \\ -0.0006 \\ 0.0032 \end{bmatrix}$$

$$u_4 := T(4) \cdot v_4 = \begin{bmatrix} -0.0004 \\ -0.0503 \\ -0.0016 \\ -0.0006 \\ -0.0801 \\ 0.0032 \end{bmatrix}$$

$$Q_4 := k_4 \cdot u_4 + Q_{f4} = \begin{bmatrix} 81.1432 \\ 41.5691 \\ 107.6386 \\ -81.1432 \\ -41.5691 \\ 141.7761 \end{bmatrix}$$

$$F_{f4} := K_4 \cdot v_4 + F_{f4} = \begin{bmatrix} -41.5691 \\ 81.1432 \\ 107.6386 \\ 41.5691 \\ -81.1432 \\ 141.7761 \end{bmatrix}$$

$$v_5 := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} 0.0503 \\ -0.0002 \\ -0.0007 \\ 0.0503 \\ -0.0004 \\ -0.0016 \end{bmatrix}$$

$$u_5 := T(5) \cdot v_5 = \begin{bmatrix} 0.0503 \\ -0.0002 \\ -0.0007 \\ 0.0503 \\ -0.0004 \\ -0.0016 \end{bmatrix}$$

$$Q_5 := k_5 \cdot u_5 + Q_{f5} = \begin{bmatrix} 13.1127 \\ 46.199 \\ -43.558 \\ -13.1127 \\ 97.801 \\ -266.0539 \end{bmatrix}$$

$$F_{f5} := K_5 \cdot v_5 + F_{f5} = \begin{bmatrix} 13.1127 \\ 46.199 \\ -43.558 \\ -13.1127 \\ 97.801 \\ -266.0539 \end{bmatrix}$$

$$v_6 := \begin{bmatrix} d_7 \\ d_8 \\ d_9 \\ d_{10} \\ d_{11} \\ d_{12} \end{bmatrix} = \begin{bmatrix} 0.0802 \\ -0.0004 \\ -0.0062 \\ 0.0801 \\ -0.0006 \\ 0.0032 \end{bmatrix}$$

$$u_6 := T(6) \cdot v_6 = \begin{bmatrix} 0.0802 \\ -0.0004 \\ -0.0062 \\ 0.0801 \\ -0.0006 \\ 0.0032 \end{bmatrix}$$

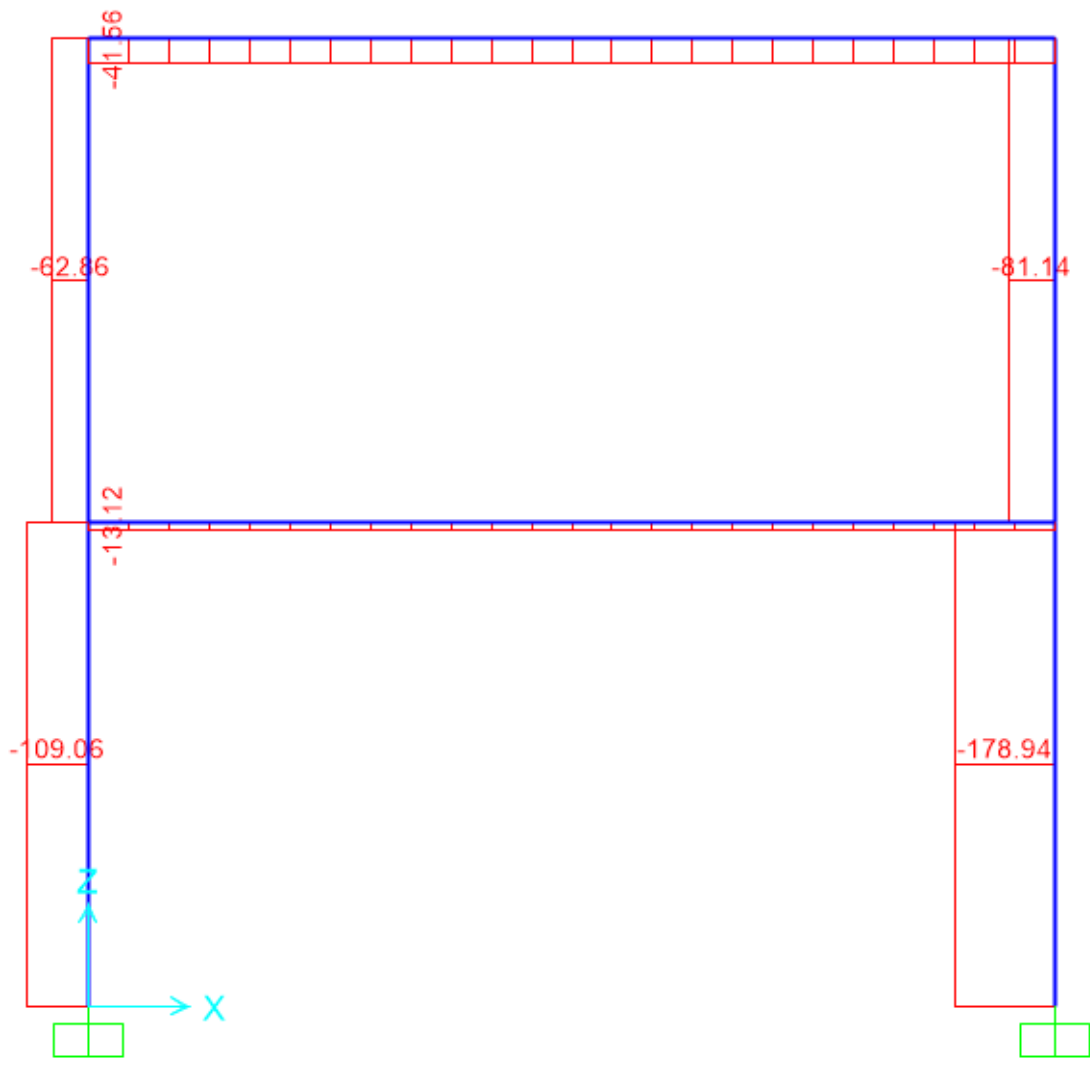
$$Q_6 := k_6 \cdot u_6 + Q_{f6} = \begin{bmatrix} 41.5691 \\ 62.8568 \\ 32.0572 \\ -41.5691 \\ 81.1432 \\ -141.7761 \end{bmatrix}$$

$$F_{f6} := K_6 \cdot v_6 + F_{f6} = \begin{bmatrix} 41.5691 \\ 62.8568 \\ 32.0572 \\ -41.5691 \\ 81.1432 \\ -141.7761 \end{bmatrix}$$

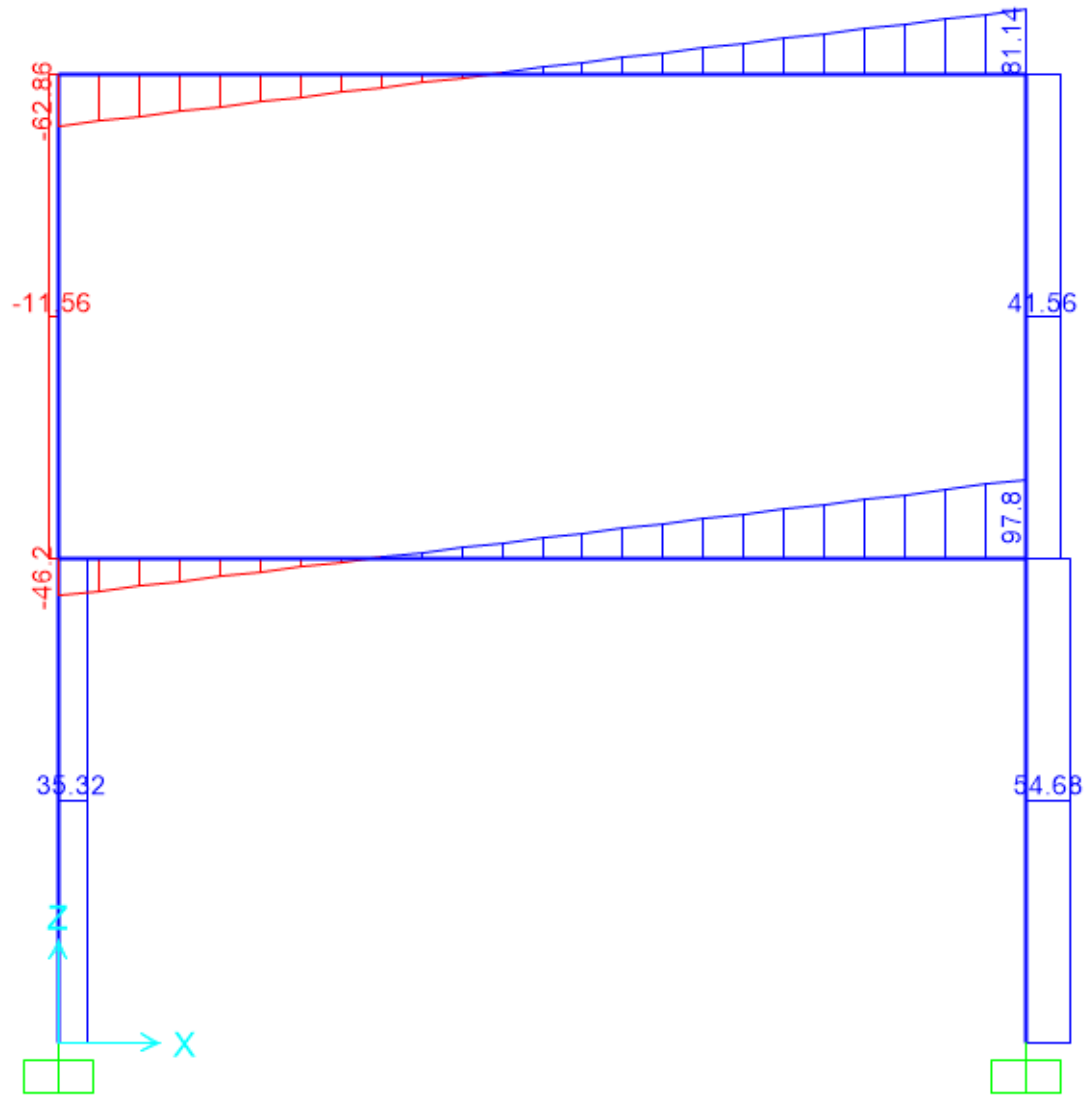
Reacciones

$$R := \begin{bmatrix} F_{f1_1} \\ F_{f1_2} \\ F_{f1_3} \\ F_{f2_1} \\ F_{f2_2} \\ F_{f2_3} \end{bmatrix} = \begin{bmatrix} -35.3182 \\ 109.0558 \\ 130.9934 \\ -54.6818 \\ 178.9442 \\ 169.6758 \end{bmatrix}$$

Fuerzas Axiales

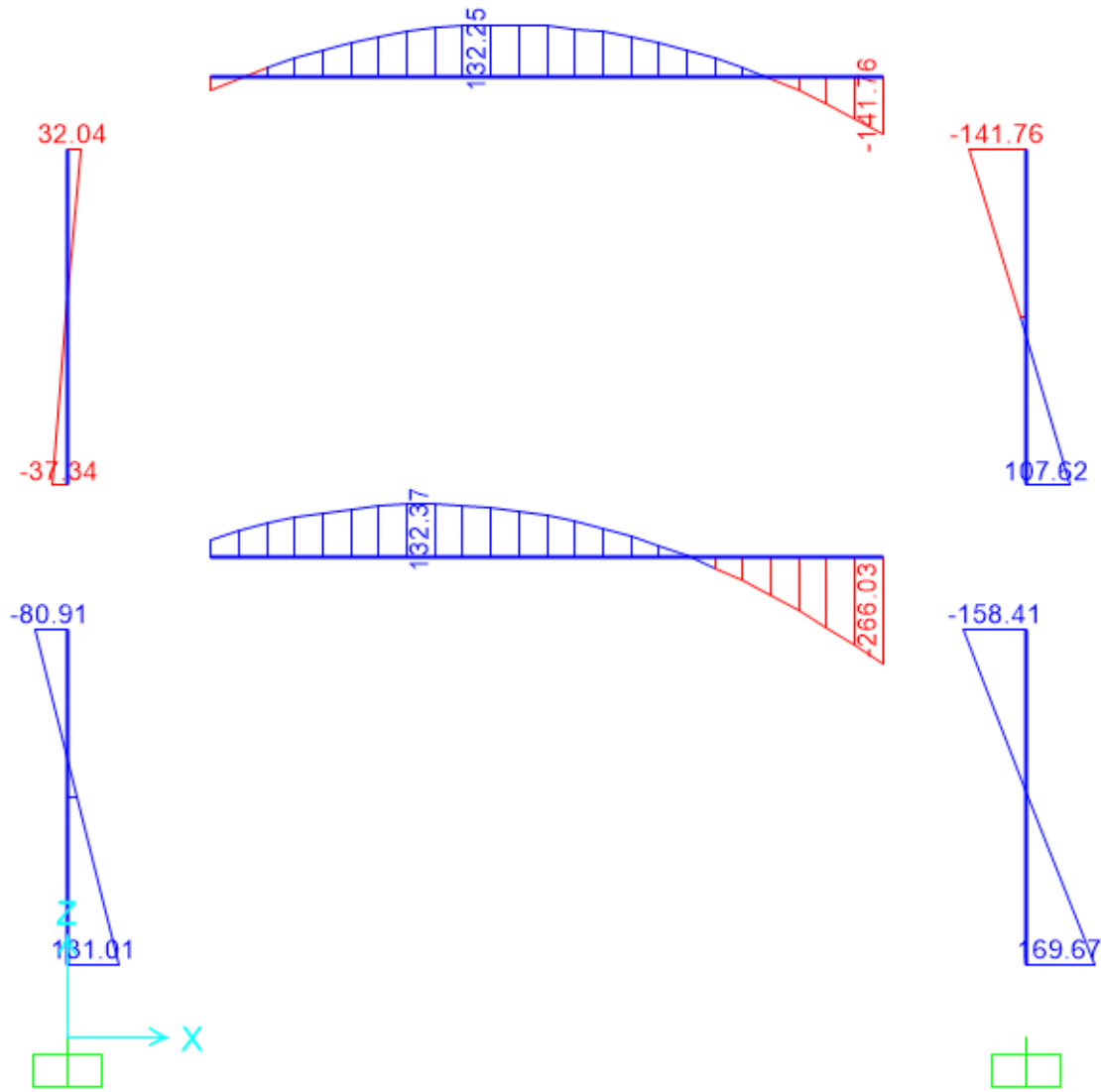


Fuerzas Cortantes





Momento Flexionante



Reacciones

