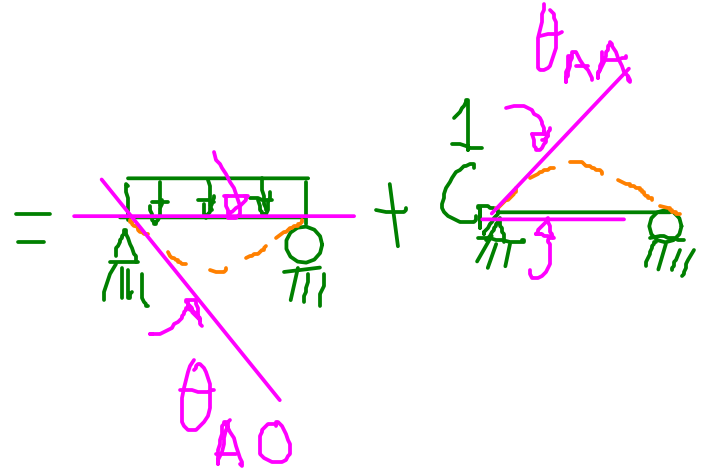
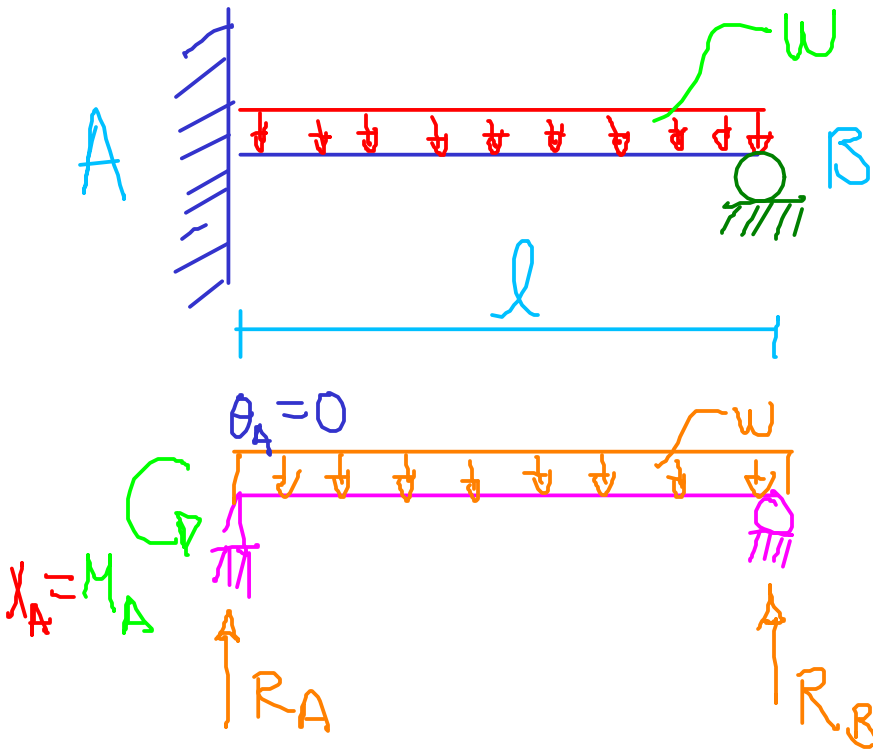


Analice la viga por el Método de las Flexibilidades. Remueva el Momento en A.

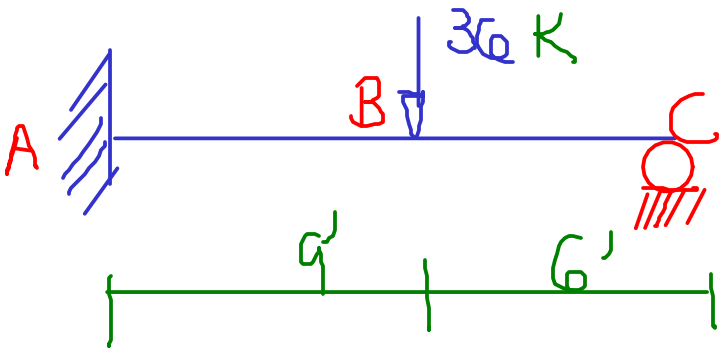


Ec. de compatibilidad:

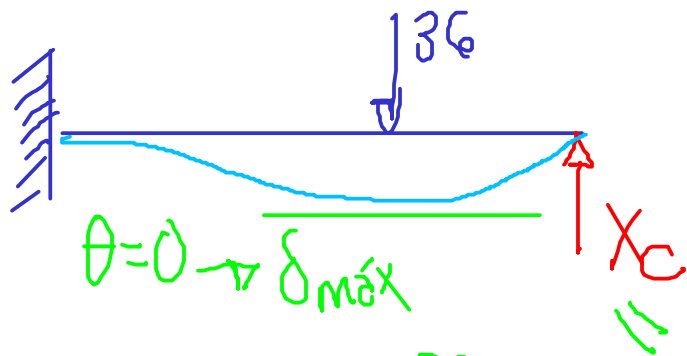
$$\theta_{AO} + \theta_{AA} X_A = 0$$

$$-\frac{wl^3}{24EI} + \frac{Ml}{3EI} X_A = 0$$

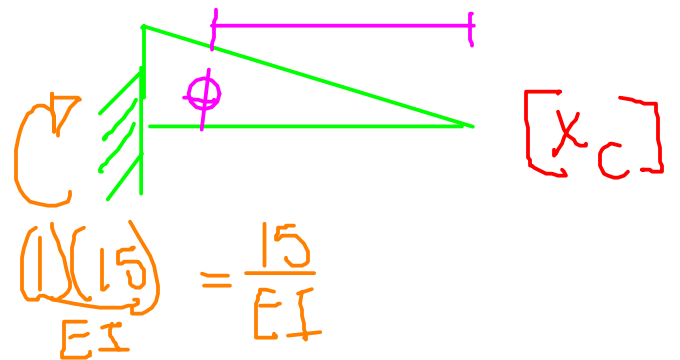
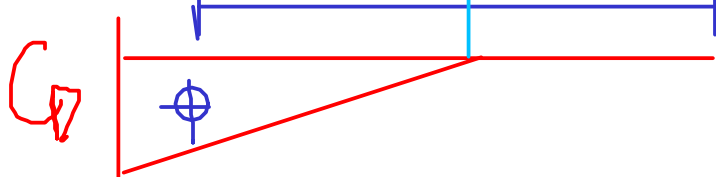
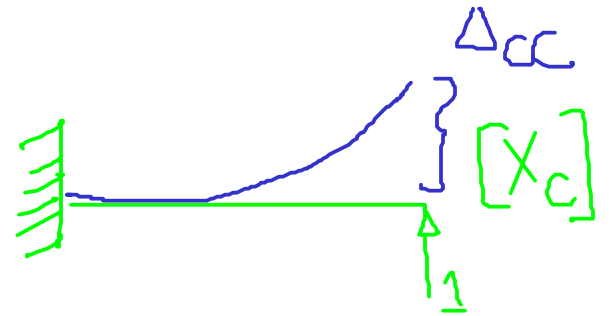
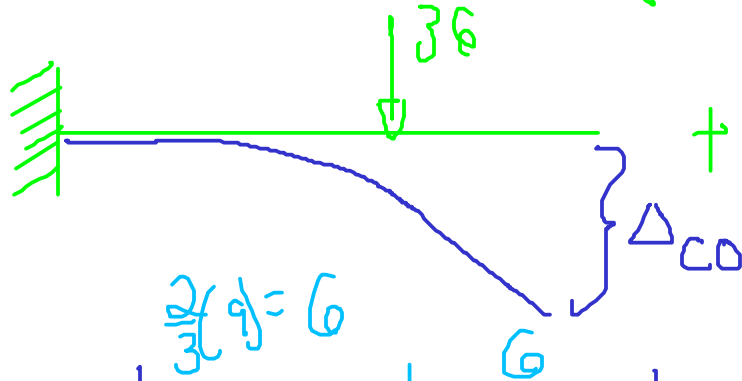
$$X_A = \frac{wl^3}{24EI} \cdot \frac{3EI}{l} = \frac{wl^2}{8} = \underline{\underline{M_A}}$$



Calcule las reacciones, dibuje las curvas de corte y momento, y ubique el punto de deflexión máxima de la viga. $EI = \text{cte}$. Obtenga la rotación en "C".



E_c compat.
 $\Delta_{co} + \Delta_{cc} X_c = 0$



$$\frac{-36(9)}{EI} = \frac{-324}{EI}$$

$$\frac{1(15)}{EI} = \frac{15}{EI}$$

$$\Delta_{co} = \frac{1}{2} (9) \left(\frac{-324}{EI} \right) \left(\frac{2}{3} \cdot 9 + 6 \right)$$

$$\Delta_{cc} = \frac{1}{2} (15) \left(\frac{15}{EI} \right) \left(\frac{2}{3} \cdot 15 \right)$$

$$\Delta_{co} = \frac{-17496}{EI}$$

$$\Delta_{cc} = \frac{1125}{EI}$$

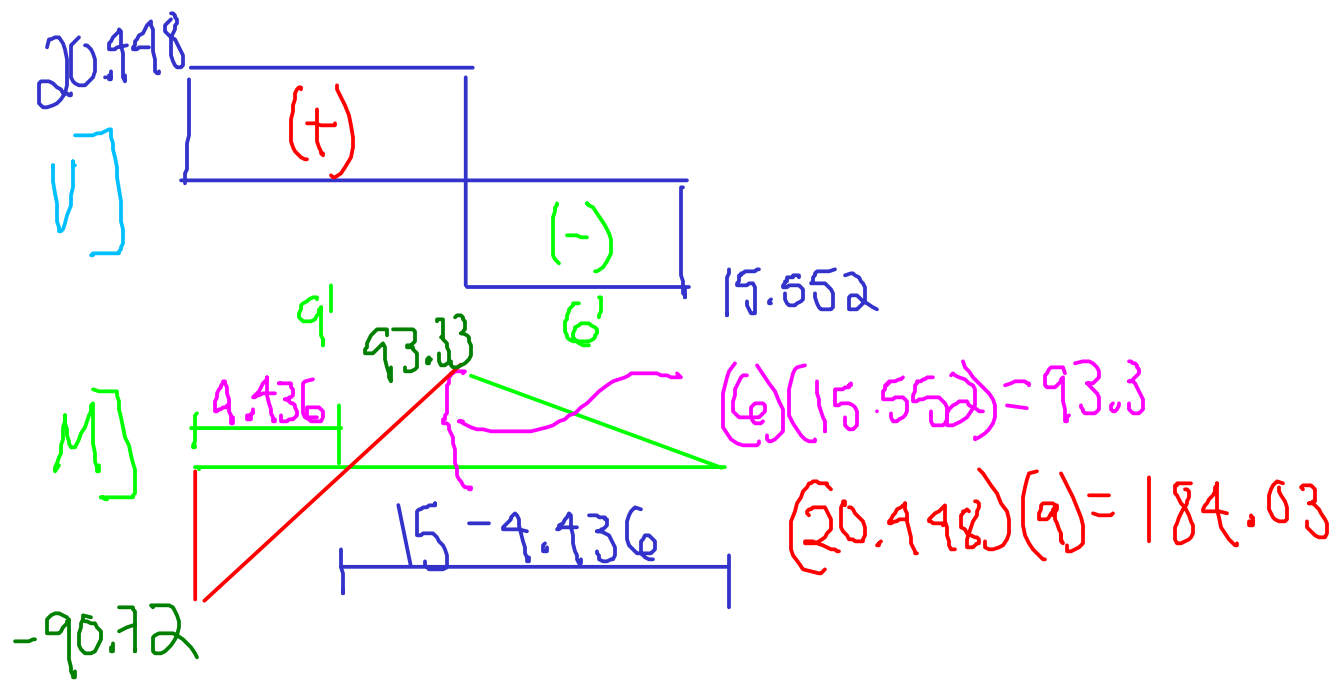
$$\Delta_{co} + \Delta_{cc} X_c = 0$$

$$\frac{-17,496}{EI} + \frac{1125 X_c}{EI} = 0$$

$$X_c = \frac{17496}{1125} = 15.55 \text{ kip.}$$

Diagramas

$$\sum F_y = 15.55 + R_A - 36 = 0 \rightarrow R_A = 20.448 \text{ kip.}$$



$$\sum M_A = \frac{-324}{EI} + \frac{15}{EI} (15.55) = -90.72$$

Rotación en "C"

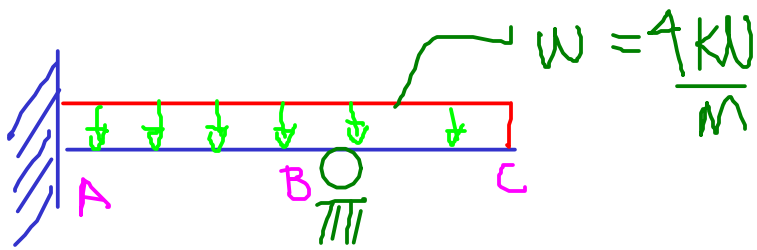
$$-90.72 + 20.448x = 0$$

$$x = 4.436$$

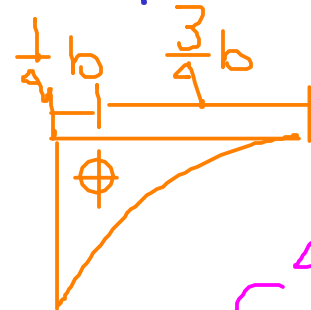
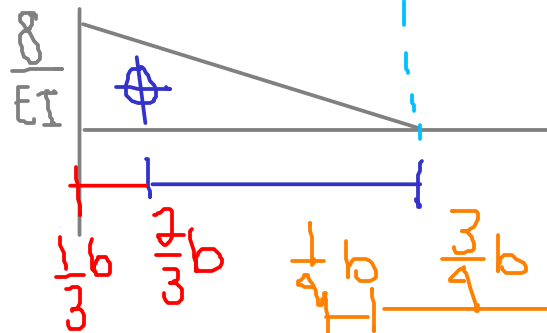
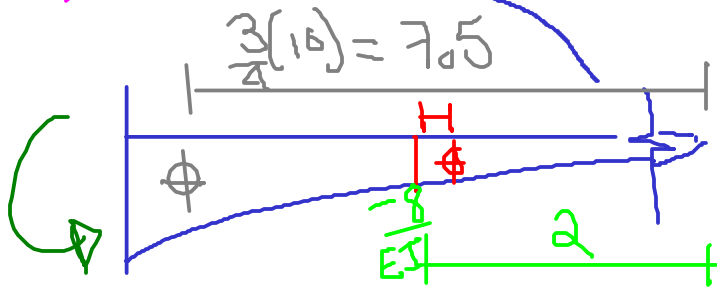
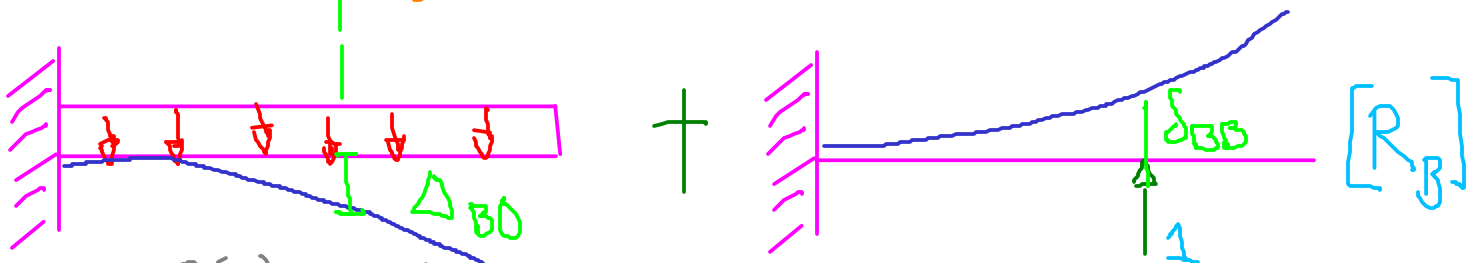
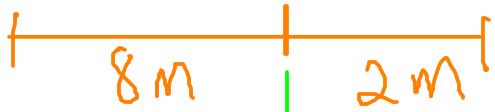
$$\theta_C = \frac{1}{2} (4.436) \left(\frac{-90.72}{EI} \right) + \frac{1}{2} (15 - 4.436) \left(\frac{93.33}{EI} \right)$$

$$= \frac{291.6}{EI}$$

$\theta = 0$ está en $2x = 2(4.436) \rightarrow \delta_{\max}$.



Para la viga, calcule las reacciones, dibuje los diagramas de corte y momento, y calcule la deflexión en "C". E del acero: 2×10^5 MPa; $I = 7490 \text{ cm}^4$.



$$(4)(10)\left(\frac{10}{2}\right) = -\frac{200}{EI}$$

Ec. de compatibilidad

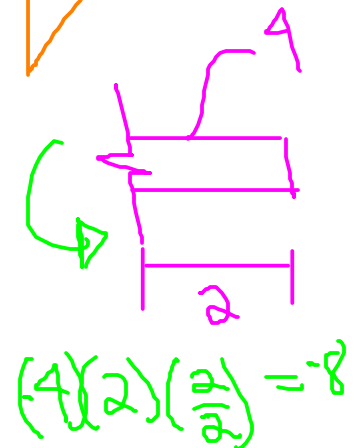
$$\Delta_{B0} + \delta_{BB} R_B = 0$$

$$\Delta_{B0} = \frac{1}{3}(10)\left(-\frac{200}{EI}\right)(7.5 - 2)$$

$$- \frac{1}{3}(2)\left(-\frac{8}{EI}\right)\left(\frac{1}{4} \cdot 2\right)$$

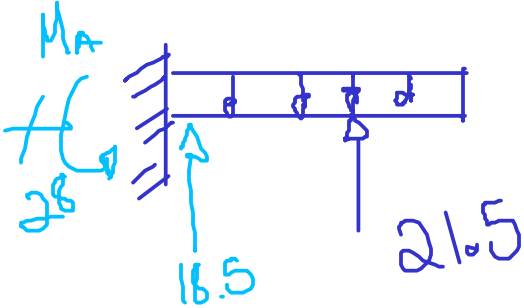
$$= -\frac{3664}{EI} \quad \downarrow$$

$$\delta_{BB} = \frac{1}{2}(8)\left(\frac{8}{EI}\right)\left(\frac{2}{3} \cdot 8\right) = \frac{512}{3EI} \quad \uparrow$$



$$-\frac{3664}{EI} + \frac{512}{3EI} R_B = 0$$

$$R_B = \frac{687}{32} = 21.5 \text{ kN} \uparrow$$

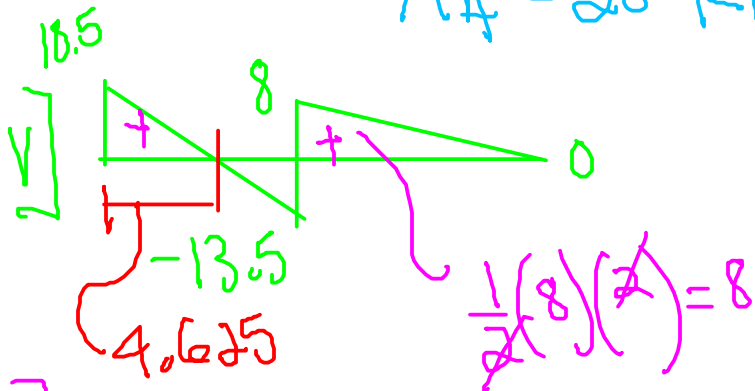


$$\sum F_y = -4(10) + 21.5 + R_A = 0$$

$$R_A = 18.5 \text{ kN} \uparrow$$

$$\sum M = (4)(10)(5) - 21.5(8) - M_A = 0$$

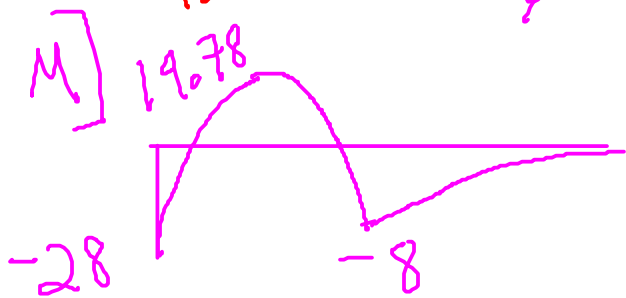
$$M_A = 28 \text{ kNm}$$



$$18.5 - 4(x) = -13.5$$

$$18.5 - 4(x) = 0$$

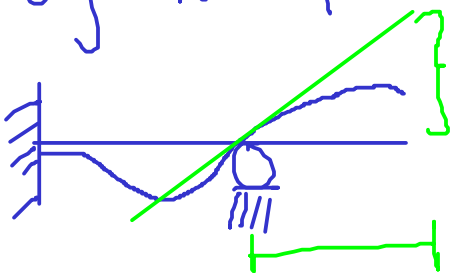
$$x = 4.625$$



$$-28 + \frac{1}{2} (4.625) (18.5) = 14.78$$

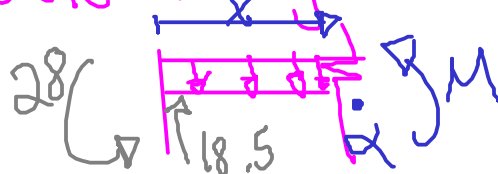
$$14.78 - \frac{1}{2} (8 - 4.625) (13.5) = -8$$

Deflexión en C



Trama AB

Doble integración θ_B



$$\Sigma M_x = 28 - 18.5x + 4(x)\left(\frac{x}{2}\right) + M = 0$$

$$M = -2x^2 + 18.5x - 28$$

$$\theta EI = \int M \quad \theta EI = -\frac{2x^3}{3} + \frac{18.5x^2}{2} - 28x + C_1$$

$$x=0 \rightarrow \theta=0$$

$$\therefore C_1 = 0.$$

$$EI\theta_B = \frac{-2(8)^3}{3} + \frac{18.5(8)^2}{2} - 28(8) \quad \begin{array}{l} M \rightarrow \text{KN}\cdot\text{m} \\ \int \rightarrow \text{KN}\cdot\text{m}^2 \end{array}$$

$$\theta_B = \frac{26.67}{EI} = \frac{26.67 \text{ KN}\cdot\text{m}^2}{(2 \times 10^5 \text{ MPa})(7490 \text{ cm}^4)}$$

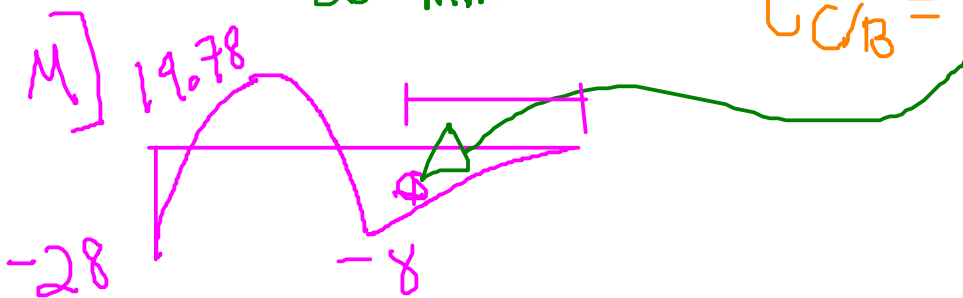
$$\frac{26.67 \text{ (1000)} \text{ (100)}^2 \text{ (100)}^2}{(2 \times 10^5) \text{ (10}^6) \text{ (7490 cm}^4)} = 0.00178 \text{ rad}$$



Área Momento Tramo BC

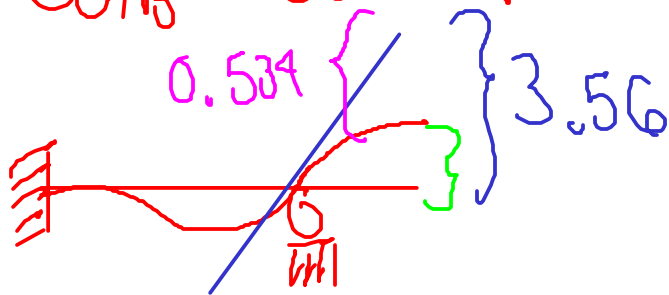
$$t_{C/B} = \frac{1}{3} (2) \left(\frac{-8}{EI}\right) \left(\frac{3}{4} \cdot 2\right)$$

MPa cm⁴

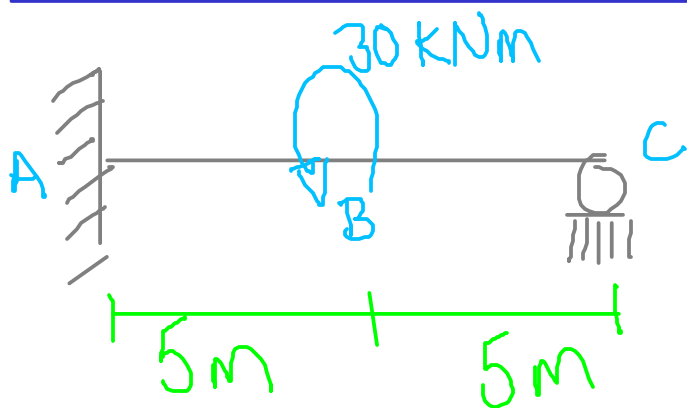


$$t_{C/B} = \frac{1}{3} \left[\frac{200 \left(-8000 \frac{\text{cm}}{\text{cm}} \right) \left(\frac{3}{4} \cdot 200 \right) \left(100 \right)^2}{2 \times 10^5 \times 10^6 \text{ N} \left(7490 \text{ cm}^4 \right)} \right]$$

$$t_{C/B} = 0.0534 \text{ cm} = 0.534 \text{ mm}$$

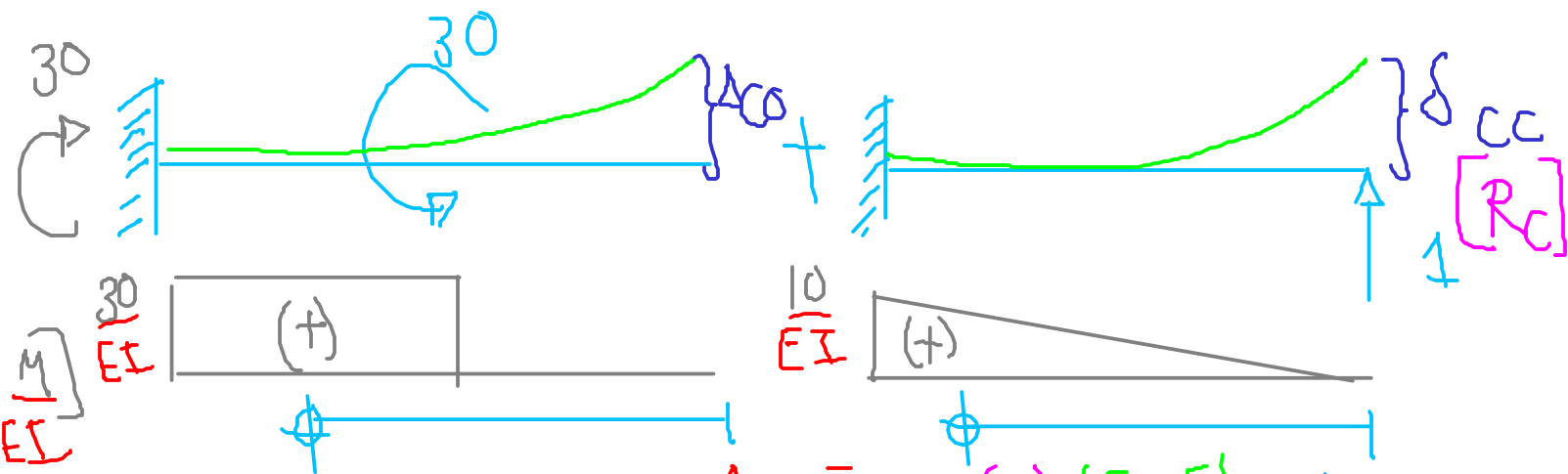


$$\delta_c = 3.56 - 0.534 = \underline{\underline{3.026 \text{ mm}}}$$



$EI = \text{constante}$

Reacciones, Corte & Momento



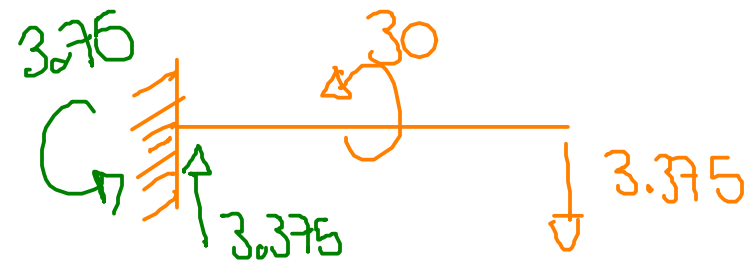
$$\Delta_{co} + \delta_{cc} R_c = 0$$

$$\Delta_{co} = \frac{30}{EI} (5) \left(\frac{5+5}{2} \right) = \frac{1125}{EI} \uparrow$$

$$\delta_{cc} = \frac{1}{2} (10) \left(\frac{10}{EI} \right) \left(\frac{2}{3} \cdot 10 \right) = \frac{1600}{3EI} \uparrow$$

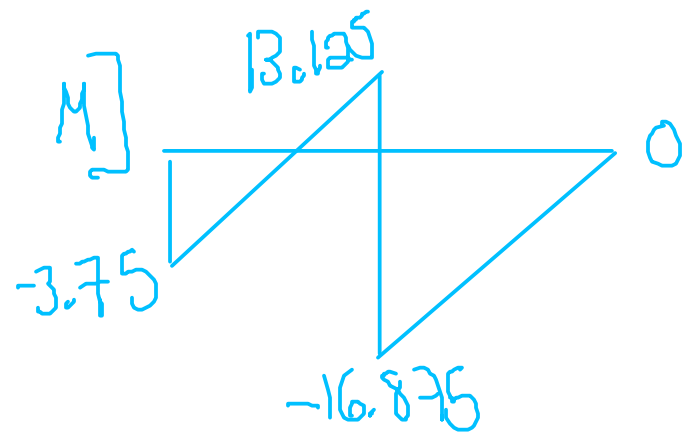
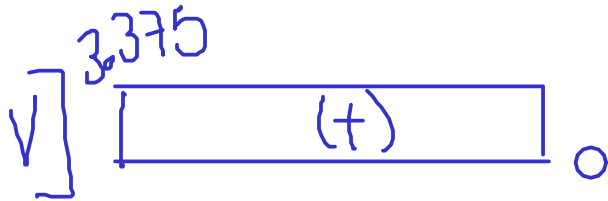
$$\frac{1125}{EI} + \frac{1000}{3EI} R_C = 0$$

$$R_C = \underline{3.375 \downarrow \text{KN}}$$



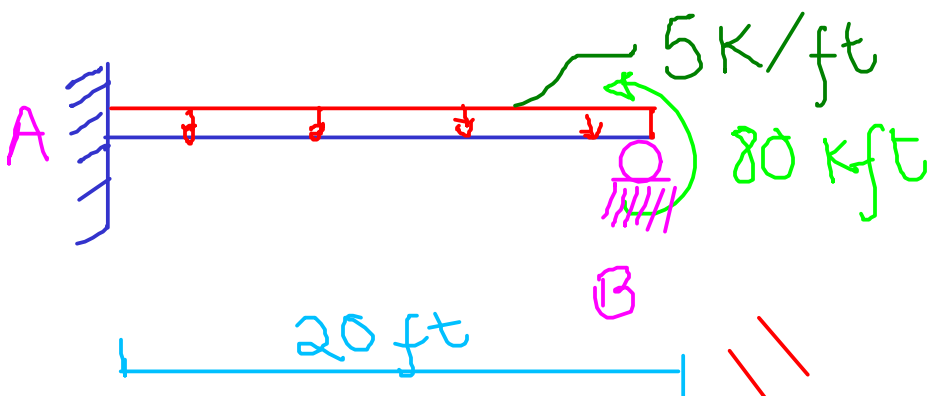
$$\sum M_A = 30 - 3.375(10) + M_A = 0$$

$$\hookrightarrow M_A = \underline{3.75}$$



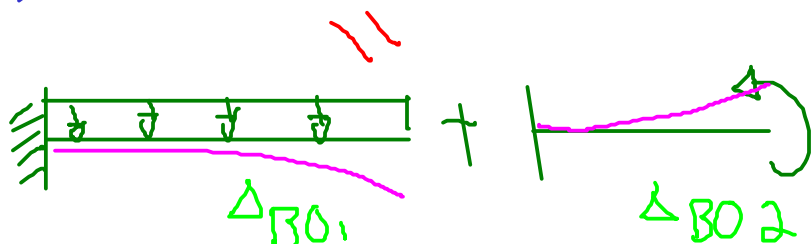
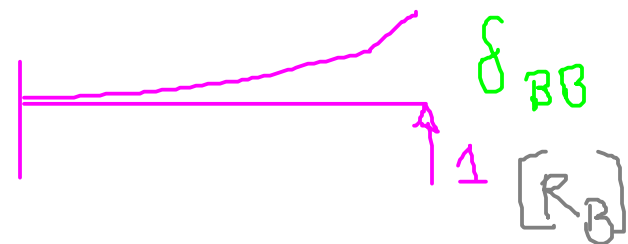
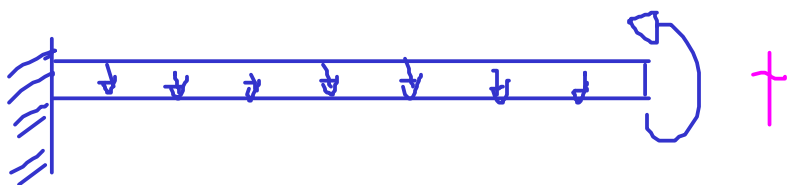
$$-3.75 + 3.375(5) = 13.125$$

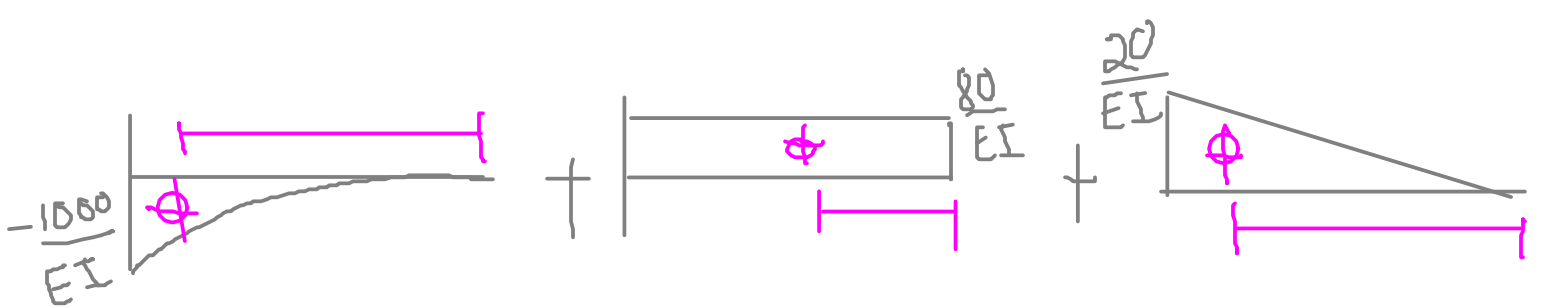
$$-16.875 + 3.375(5) = 0$$



Reacciones,
Corte & Momento

$EI = \text{constante}$





$$(5)(20)\left(\frac{20}{2}\right) = \frac{-1000}{EI}$$

$$\Delta_{BO_1} = \frac{1}{3}(20)\left(\frac{-1000}{EI}\right)\left(\frac{3 \cdot 20}{4}\right) = \frac{-100,000}{EI} \downarrow$$

$$\Delta_{BO_2} = \left(\frac{80}{EI}\right)(20)(10) = \frac{16,000}{EI} \uparrow$$

$$\delta_{BB} = \frac{1}{2}(20)\left(\frac{20}{EI}\right)\left(\frac{2}{3} \cdot 20\right) = \frac{8,000}{3EI} \uparrow$$

Ecuación de compatibilidad

$$\Delta_{BO_1} + \Delta_{BO_2} + \delta_{BB} R_B = 0$$

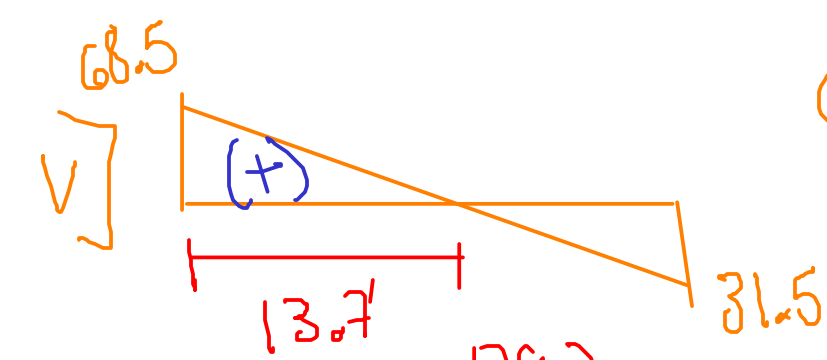
$$\frac{-100,000}{EI} + \frac{16,000}{EI} + \frac{8,000}{3EI} R_B = 0$$

$$R_B = 31.5 \text{ Kip.}$$

$$\sum F_y = 0 = 31.5 + R_A - 5(20) = 0 \rightarrow R_A = 68.5$$

$$\sum M_A = M_A + 80 - 5(20)(10) + 31.5(20) = 0$$

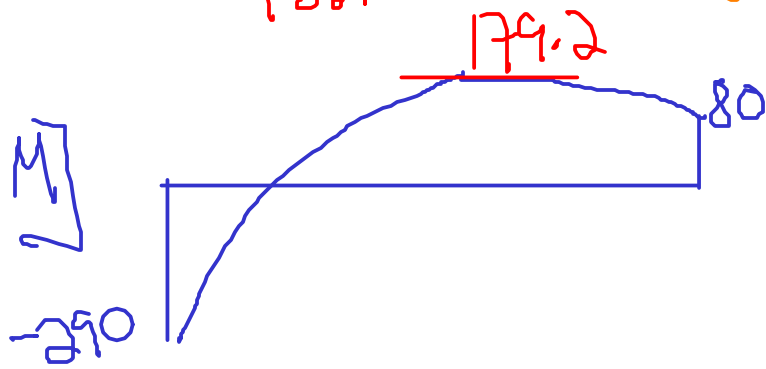
$$\rightarrow M_A = 290$$



$$68.5 - 5(20) = -31.5$$

$$68.5 - 5x = 0$$

$$x = 13.7$$



$$-290 + \frac{1}{2}(13.7)(68.5)$$

$$= 179.2$$

$$179.2 - \frac{1}{2}(20 - 13.7)(31.5)$$

$$= +80$$