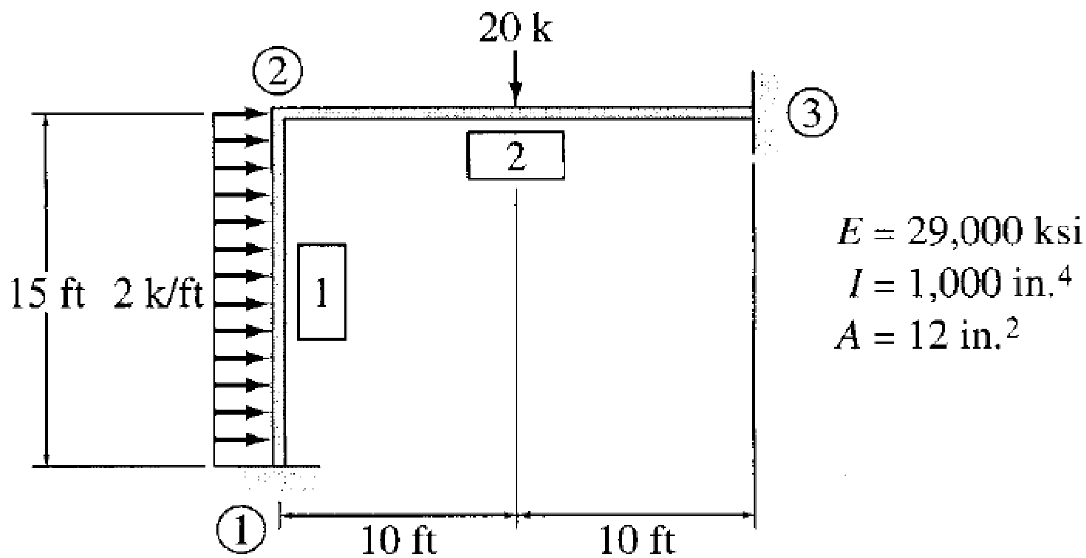
Ejemplo del Método General de las Rigideces, aplicado en Marcos



Encuentre las reacciones y las fuerzas en los extremos de los miembros en las coordenadas locales para el marco mostrado, empleando el método matricial de las rigideces.

$L_1 := 15$ ft $\qquad E_1 := 29000 \cdot 12^2$ ksf $\qquad coseno_1 := 0 \qquad seno_1 := 1$

$A_1 := \dfrac{12}{12^2}$ ft^2 $\qquad I_1 := \dfrac{1000}{12^4}$ ft^4

Matriz de rigidez local del miembro 1:

$$k_1 := \frac{E_1 \cdot I_1}{L_1^3} \begin{pmatrix} \frac{A_1 \cdot L_1^2}{I_1} & 0 & 0 & \frac{-A_1 \cdot L_1^2}{I_1} & 0 & 0 \\ 0 & 12 & 6L_1 & 0 & -12 & 6L_1 \\ 0 & 6L_1 & 4L_1^2 & 0 & -6L_1 & 2L_1^2 \\ \frac{-A_1 \cdot L_1^2}{I_1} & 0 & 0 & \frac{A_1 \cdot L_1^2}{I_1} & 0 & 0 \\ 0 & -12 & -6L_1 & 0 & 12 & -6L_1 \\ 0 & 6L_1 & 2L_1^2 & 0 & -6L_1 & 4L_1^2 \end{pmatrix}$$

$$k_1 = \begin{pmatrix} 2.32\times10^4 & 0 & 0 & -2.32\times10^4 & 0 & 0 \\ 0 & 716.049 & 5.37\times10^3 & 0 & -716.049 & 5.37\times10^3 \\ 0 & 5.37\times10^3 & 5.37\times10^4 & 0 & -5.37\times10^3 & 2.685\times10^4 \\ -2.32\times10^4 & 0 & 0 & 2.32\times10^4 & 0 & 0 \\ 0 & -716.049 & -5.37\times10^3 & 0 & 716.049 & -5.37\times10^3 \\ 0 & 5.37\times10^3 & 2.685\times10^4 & 0 & -5.37\times10^3 & 5.37\times10^4 \end{pmatrix}$$

Vector de fuerzas locales, Q.

$$w := 2 \qquad M_{empExtr1} := \frac{w\cdot L_1{}^2}{12} = 37.5 \qquad V_1 := \frac{w\cdot L_1}{2} = 15 \qquad N_1 := 0$$

$$Q_{f1} := \begin{pmatrix} N_1 \\ V_1 \\ M_{empExtr1} \\ N_1 \\ V_1 \\ -M_{empExtr1} \end{pmatrix} = \begin{pmatrix} 0 \\ 15 \\ 37.5 \\ 0 \\ 15 \\ -37.5 \end{pmatrix}$$

$$T_1 := \begin{pmatrix} coseno_1 & seno_1 & 0 & 0 & 0 & 0 \\ -seno_1 & coseno_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & coseno_1 & seno_1 & 0 \\ 0 & 0 & 0 & -seno_1 & coseno_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F_{f1} := T_1{}^{-1}\cdot Q_{f1} = \begin{pmatrix} -15 \\ 0 \\ 37.5 \\ -15 \\ 0 \\ -37.5 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

Vector de fuerzas globales, F.

$$(0\ \ 0\ \ 0\ \ 1\ \ 2\ \ 3)$$

$$K_1 := {T_1}^T \cdot k_1 \cdot T_1 = \begin{pmatrix} 716.049 & 0 & -5.37\times10^3 & -716.049 & 0 & -5.37\times10^3 \\ 0 & 2.32\times10^4 & 0 & 0 & -2.32\times10^4 & 0 \\ -5.37\times10^3 & 0 & 5.37\times10^4 & 5.37\times10^3 & 0 & 2.685\times10^4 \\ -716.049 & 0 & 5.37\times10^3 & 716.049 & 0 & 5.37\times10^3 \\ 0 & -2.32\times10^4 & 0 & 0 & 2.32\times10^4 & 0 \\ -5.37\times10^3 & 0 & 2.685\times10^4 & 5.37\times10^3 & 0 & 5.37\times10^4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

Miembro 2

$L_2 := 20$ ft $\quad E_2 := 29000\cdot 12^2 \quad coseno_2 := 1 \quad seno_2 := 0$

$A_2 := \dfrac{12}{12^2}$ ft^2 $\quad I_2 := \dfrac{1000}{12^4}$

Matriz de rigidez local del miembro 2:

$$k_2 := \frac{E_2\cdot I_2}{{L_2}^3} \begin{pmatrix} \frac{A_2\cdot {L_2}^2}{I_2} & 0 & 0 & \frac{-A_2\cdot {L_2}^2}{I_2} & 0 & 0 \\ 0 & 12 & 6L_2 & 0 & -12 & 6L_2 \\ 0 & 6L_2 & 4{L_2}^2 & 0 & -6L_2 & 2{L_2}^2 \\ \frac{-A_2\cdot {L_2}^2}{I_2} & 0 & 0 & \frac{A_2\cdot {L_2}^2}{I_2} & 0 & 0 \\ 0 & -12 & -6L_2 & 0 & 12 & -6L_2 \\ 0 & 6L_2 & 2{L_2}^2 & 0 & -6L_2 & 4{L_2}^2 \end{pmatrix}$$

$$(1\ \ 2\ \ 3\ \ 0\ \ 0\ \ 0)$$

$$k_2 = \begin{pmatrix} 1.74\times10^4 & 0 & 0 & -1.74\times10^4 & 0 & 0 \\ 0 & 302.083 & 3.021\times10^3 & 0 & -302.083 & 3.021\times10^3 \\ 0 & 3.021\times10^3 & 4.028\times10^4 & 0 & -3.021\times10^3 & 2.014\times10^4 \\ -1.74\times10^4 & 0 & 0 & 1.74\times10^4 & 0 & 0 \\ 0 & -302.083 & -3.021\times10^3 & 0 & 302.083 & -3.021\times10^3 \\ 0 & 3.021\times10^3 & 2.014\times10^4 & 0 & -3.021\times10^3 & 4.028\times10^4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$P := 20$ $\qquad M_{empExtr2} := \frac{P \cdot L_2}{8} = 50$ $\qquad V_2 := \frac{P}{2} = 10$ $\qquad N_2 := 0$

$$Q_{f2} := \begin{pmatrix} N_2 \\ V_2 \\ M_{empExtr2} \\ N_2 \\ V_2 \\ -M_{empExtr2} \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 50 \\ 0 \\ 10 \\ -50 \end{pmatrix}$$

Vector de fuerzas locales, Q.

$$T_2 := \begin{pmatrix} coseno_2 & seno_2 & 0 & 0 & 0 & 0 \\ -seno_2 & coseno_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & coseno_2 & seno_2 & 0 \\ 0 & 0 & 0 & -seno_2 & coseno_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F_{f2} := T_2^{-1} \cdot Q_{f2} = \begin{pmatrix} 0 \\ 10 \\ 50 \\ 0 \\ 10 \\ -50 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Vector de fuerzas globales, F.

$$K_2 := T_2^{T} \cdot k_2 \cdot T_2$$

$$\begin{pmatrix} 1 & 2 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 1.74 \times 10^4 & 0 & 0 & -1.74 \times 10^4 & 0 & 0 \\ 0 & 302.083 & 3.021 \times 10^3 & 0 & -302.083 & 3.021 \times 10^3 \\ 0 & 3.021 \times 10^3 & 4.028 \times 10^4 & 0 & -3.021 \times 10^3 & 2.014 \times 10^4 \\ -1.74 \times 10^4 & 0 & 0 & 1.74 \times 10^4 & 0 & 0 \\ 0 & -302.083 & -3.021 \times 10^3 & 0 & 302.083 & -3.021 \times 10^3 \\ 0 & 3.021 \times 10^3 & 2.014 \times 10^4 & 0 & -3.021 \times 10^3 & 4.028 \times 10^4 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Matriz de rigidez global del sistema:

$$S := \begin{pmatrix} K1_{4,4} + K2_{1,1} & K1_{4,5} + K2_{1,2} & K1_{4,6} + K2_{1,3} \\ K1_{5,4} + K2_{2,1} & K1_{5,5} + K2_{2,2} & K1_{5,6} + K2_{2,3} \\ K1_{6,4} + K2_{3,1} & K1_{6,5} + K2_{3,2} & K1_{6,6} + K2_{3,3} \end{pmatrix} = \begin{pmatrix} 1.812 \times 10^{4} & 0 & 5.37 \times 10^{3} \\ 0 & 2.35 \times 10^{4} & 3.021 \times 10^{3} \\ 5.37 \times 10^{3} & 3.021 \times 10^{3} & 9.398 \times 10^{4} \end{pmatrix}$$

$$P := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad P_f := \begin{pmatrix} Ff1_4 + Ff2_1 \\ Ff1_5 + Ff2_2 \\ Ff1_6 + Ff2_3 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 12.5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

P es el vector de cargas en los nodos.

Pf es el vector de fuerzas de nodo empotrado de la estructura

Solución del sistema matricial:

P-Pf = Sd

$$d := S^{-1} \cdot (P - P_f) = \begin{pmatrix} 8.785 \times 10^{-4} \\ -4.036 \times 10^{-4} \\ -1.702 \times 10^{-4} \end{pmatrix}$$

Fuerzas en los miembros

$$v_1 := \begin{pmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 8.785 \times 10^{-4} \\ -4.036 \times 10^{-4} \\ -1.702 \times 10^{-4} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

$$u_1 := T_1 \cdot v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -4.036 \times 10^{-4} \\ -8.785 \times 10^{-4} \\ -1.702 \times 10^{-4} \end{pmatrix}$$

$$Q_1 := k_1 \cdot u_1 + Q_{f1} = \begin{pmatrix} 9.364 \\ 14.715 \\ 37.647 \\ -9.364 \\ 15.285 \\ -41.924 \end{pmatrix}$$

$$F_1 := K_1 \cdot v_1 + F_{f1} = \begin{pmatrix} -14.715 \\ 9.364 \\ 37.647 \\ -15.285 \\ -9.364 \\ -41.924 \end{pmatrix}$$

$$v_2 := \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 8.785 \times 10^{-4} \\ -4.036 \times 10^{-4} \\ -1.702 \times 10^{-4} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$u_2 := v_2 = \begin{pmatrix} 8.785 \times 10^{-4} \\ -4.036 \times 10^{-4} \\ -1.702 \times 10^{-4} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Q_2 := k_2 \cdot u_2 + Q_{f2} = \begin{pmatrix} 15.285 \\ 9.364 \\ 41.924 \\ -15.285 \\ 10.636 \\ -54.647 \end{pmatrix}$$

$$F_2 := Q_2 = \begin{pmatrix} 15.285 \\ 9.364 \\ 41.924 \\ -15.285 \\ 10.636 \\ -54.647 \end{pmatrix}$$