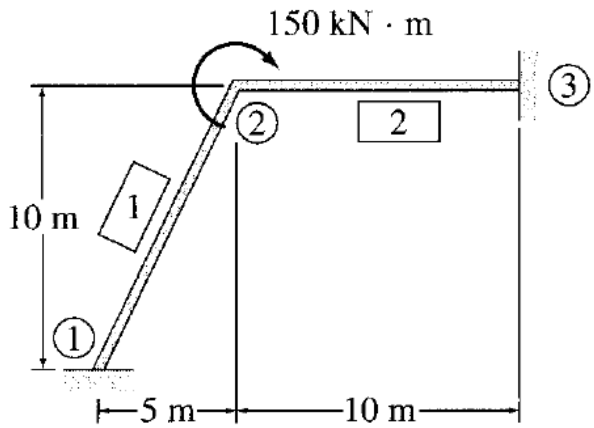
Ejemplo del Método General de las Rigideces, aplicado en Marcos



$E = 200$ GPa

$I = 400\ (10^6)\ \text{mm}^4$

$A = 4{,}000\ \text{mm}^2$

Encuentre las reacciones y las fuerzas en los extremos de los miembros en las coordenadas locales para el marco mostrado, empleando el método matricial de las rigideces.

$L_1 := \sqrt{5^2 + 10^2} = 11.18$ m $\qquad E_1 := 200 \cdot 10^6$ kPa $\qquad coseno_1 := \frac{5}{\sqrt{5^2 + 10^2}} = 0.447$

$A_1 := \frac{4000}{1000^2}$ m^2 $\qquad I_1 := \frac{400 \cdot 10^6}{1000^4}$ m^4 $\qquad seno_1 := \frac{10}{\sqrt{5^2 + 10^2}} = 0.894$

Matriz de rigidez local del miembro 1:

$$k_1 := \frac{E_1 \cdot I_1}{L_1^3} \begin{pmatrix} \frac{A_1 \cdot L_1^2}{I_1} & 0 & 0 & \frac{-A_1 \cdot L_1^2}{I_1} & 0 & 0 \\ 0 & 12 & 6L_1 & 0 & -12 & 6L_1 \\ 0 & 6L_1 & 4L_1^2 & 0 & -6L_1 & 2L_1^2 \\ \frac{-A_1 \cdot L_1^2}{I_1} & 0 & 0 & \frac{A_1 \cdot L_1^2}{I_1} & 0 & 0 \\ 0 & -12 & -6L_1 & 0 & 12 & -6L_1 \\ 0 & 6L_1 & 2L_1^2 & 0 & -6L_1 & 4L_1^2 \end{pmatrix}$$

$$k_1 = \begin{pmatrix} 7.155\times10^4 & 0 & 0 & -7.155\times10^4 & 0 & 0 \\ 0 & 686.92 & 3.84\times10^3 & 0 & -686.92 & 3.84\times10^3 \\ 0 & 3.84\times10^3 & 2.862\times10^4 & 0 & -3.84\times10^3 & 1.431\times10^4 \\ -7.155\times10^4 & 0 & 0 & 7.155\times10^4 & 0 & 0 \\ 0 & -686.92 & -3.84\times10^3 & 0 & 686.92 & -3.84\times10^3 \\ 0 & 3.84\times10^3 & 1.431\times10^4 & 0 & -3.84\times10^3 & 2.862\times10^4 \end{pmatrix}$$

Vector de fuerzas locales, Q.

No aplica.

$w := 0 \qquad M_{empExtr1} := 0 \qquad V_1 := 0 \qquad N_1 := 0$

$$Q_{f1} := \begin{pmatrix} N_1 \\ V_1 \\ M_{empExtr1} \\ N_1 \\ V_1 \\ -M_{empExtr1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$T_1 := \begin{pmatrix} coseno_1 & seno_1 & 0 & 0 & 0 & 0 \\ -seno_1 & coseno_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & coseno_1 & seno_1 & 0 \\ 0 & 0 & 0 & -seno_1 & coseno_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.447 & 0.894 & 0 & 0 & 0 & 0 \\ -0.894 & 0.447 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.447 & 0.894 & 0 \\ 0 & 0 & 0 & -0.894 & 0.447 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F_{f1} := T_1^{-1} \cdot Q_{f1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

Vector de fuerzas globales, F.

No aplica

$(0\ \ 0\ \ 0\ \ 1\ \ 2\ \ 3)$

$$K_1 := T_1^{\,T}\cdot k_1\cdot T_1 = \begin{pmatrix} 14860.371 & 28346.902 & -3434.6 & -14860.371 & -28346.902 & -3434.6 \\ 28346.902 & 57380.724 & 1717.3 & -28346.902 & -57380.724 & 1717.3 \\ -3434.6 & 1717.3 & 28621.67 & 3434.6 & -1717.3 & 14310.835 \\ -14860.371 & -28346.902 & 3434.6 & 14860.371 & 28346.902 & 3434.6 \\ -28346.902 & -57380.724 & -1717.3 & 28346.902 & 57380.724 & -1717.3 \\ -3434.6 & 1717.3 & 14310.835 & 3434.6 & -1717.3 & 28621.67 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

Miembro 2

$L_2 := 10$ m $\qquad E_2 := 200\cdot 10^6$ kPa $\qquad \text{coseno}_2 := \dfrac{10}{10} = 1$

$A_2 := \dfrac{4000}{1000^2}$ m^2 $\qquad I_2 := \dfrac{400\times 10^6}{1000^4}$ m^4 $\qquad \text{seno}_2 := \dfrac{0}{10} = 0$

Matriz de rigidez local del miembro 2:

$$k_2 := \frac{E_2\cdot I_2}{L_2^{\,3}} \begin{pmatrix} \dfrac{A_2\cdot L_2^{\,2}}{I_2} & 0 & 0 & \dfrac{-A_2\cdot L_2^{\,2}}{I_2} & 0 & 0 \\ 0 & 12 & 6L_2 & 0 & -12 & 6L_2 \\ 0 & 6L_2 & 4L_2^{\,2} & 0 & -6L_2 & 2L_2^{\,2} \\ \dfrac{-A_2\cdot L_2^{\,2}}{I_2} & 0 & 0 & \dfrac{A_2\cdot L_2^{\,2}}{I_2} & 0 & 0 \\ 0 & -12 & -6L_2 & 0 & 12 & -6L_2 \\ 0 & 6L_2 & 2L_2^{\,2} & 0 & -6L_2 & 4L_2^{\,2} \end{pmatrix}$$

$(1\ \ 2\ \ 3\ \ 0\ \ 0\ \ 0)$

$$k_2 = \begin{pmatrix} 8\times 10^4 & 0 & 0 & -8\times 10^4 & 0 & 0 \\ 0 & 960 & 4.8\times 10^3 & 0 & -960 & 4.8\times 10^3 \\ 0 & 4.8\times 10^3 & 3.2\times 10^4 & 0 & -4.8\times 10^3 & 1.6\times 10^4 \\ -8\times 10^4 & 0 & 0 & 8\times 10^4 & 0 & 0 \\ 0 & -960 & -4.8\times 10^3 & 0 & 960 & -4.8\times 10^3 \\ 0 & 4.8\times 10^3 & 1.6\times 10^4 & 0 & -4.8\times 10^3 & 3.2\times 10^4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$P := 0$ $M_{empExtr2} := 0$ $V_2 := 0$ $N_2 := 0$

No aplica.

$$Q_{f2} := \begin{pmatrix} N_2 \\ V_2 \\ M_{empExtr2} \\ N_2 \\ V_2 \\ -M_{empExtr2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Vector de fuerzas locales, Q.

$$T_2 := \begin{pmatrix} coseno_2 & seno_2 & 0 & 0 & 0 & 0 \\ -seno_2 & coseno_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & coseno_2 & seno_2 & 0 \\ 0 & 0 & 0 & -seno_2 & coseno_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F_{f2} := T_2^{-1} \cdot Q_{f2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Vector de fuerzas globales, F.

$$K_2 := T_2^{T} \cdot k_2 \cdot T_2$$

$$\begin{pmatrix} 1 & 2 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 8 \times 10^4 & 0 & 0 & -8 \times 10^4 & 0 & 0 \\ 0 & 960 & 4.8 \times 10^3 & 0 & -960 & 4.8 \times 10^3 \\ 0 & 4.8 \times 10^3 & 3.2 \times 10^4 & 0 & -4.8 \times 10^3 & 1.6 \times 10^4 \\ -8 \times 10^4 & 0 & 0 & 8 \times 10^4 & 0 & 0 \\ 0 & -960 & -4.8 \times 10^3 & 0 & 960 & -4.8 \times 10^3 \\ 0 & 4.8 \times 10^3 & 1.6 \times 10^4 & 0 & -4.8 \times 10^3 & 3.2 \times 10^4 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Matriz de rigidez global del sistema:

$$S := \begin{pmatrix} K1_{4,4} + K2_{1,1} & K1_{4,5} + K2_{1,2} & K1_{4,6} + K2_{1,3} \\ K1_{5,4} + K2_{2,1} & K1_{5,5} + K2_{2,2} & K1_{5,6} + K2_{2,3} \\ K1_{6,4} + K2_{3,1} & K1_{6,5} + K2_{3,2} & K1_{6,6} + K2_{3,3} \end{pmatrix} = \begin{pmatrix} 9.486 \times 10^{4} & 2.835 \times 10^{4} & 3.435 \times 10^{3} \\ 2.835 \times 10^{4} & 5.834 \times 10^{4} & 3.083 \times 10^{3} \\ 3.435 \times 10^{3} & 3.083 \times 10^{3} & 6.062 \times 10^{4} \end{pmatrix}$$

$$P := \begin{pmatrix} 0 \\ 0 \\ -150 \end{pmatrix} \quad P_f := \begin{pmatrix} F_{f1_4} + F_{f2_1} \\ F_{f1_5} + F_{f2_2} \\ F_{f1_6} + F_{f2_3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

P es el vector de cargas en los nodos.

Pf es el vector de fuerzas de nodo empotrado de la estructura

Solución del sistema matricial:

P-Pf = Sd

$$d := S^{-1} \cdot (P - P_f) = \begin{pmatrix} 5.93 \times 10^{-5} \\ 1.024 \times 10^{-4} \\ -2.483 \times 10^{-3} \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Fuerzas en los miembros

$$v_1 := \begin{pmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5.93 \times 10^{-5} \\ 1.024 \times 10^{-4} \\ -2.483 \times 10^{-3} \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

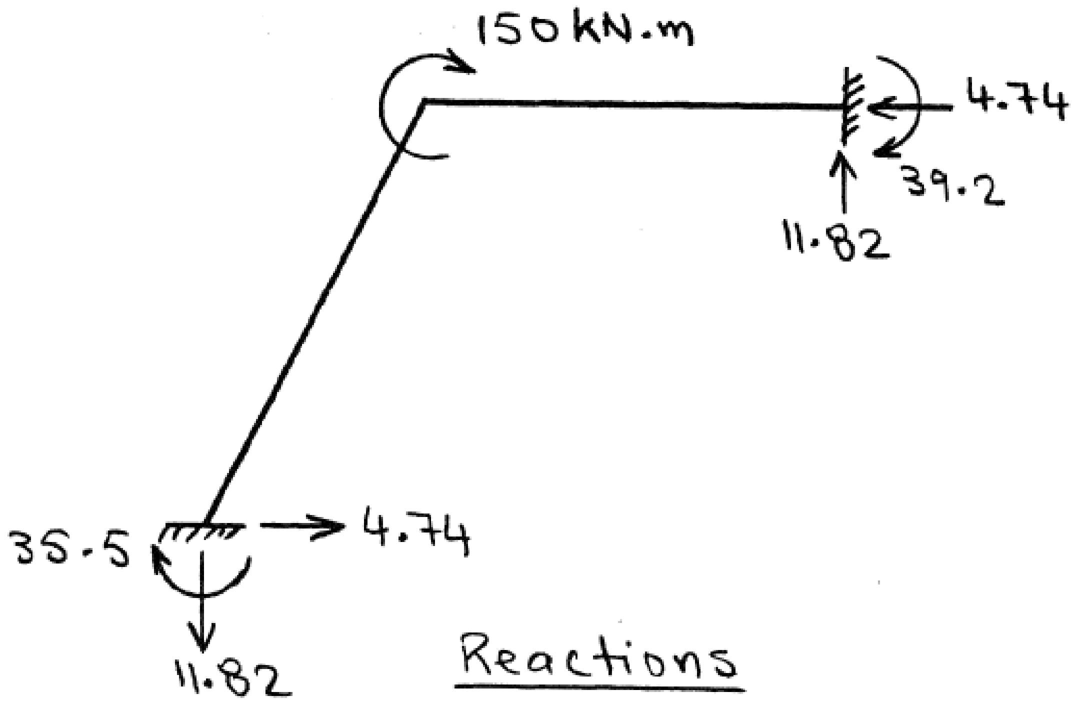
$$u_1 := T_1 \cdot v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.181 \times 10^{-4} \\ -7.257 \times 10^{-6} \\ -2.483 \times 10^{-3} \end{pmatrix}$$

$$Q_1 := k_1 \cdot u_1 + Q_{f1} = \begin{pmatrix} -8.45 \\ -9.529 \\ -35.505 \\ 8.45 \\ 9.529 \\ -71.038 \end{pmatrix}$$

$$F_1 := K_1 \cdot v_1 + F_{f1} = \begin{pmatrix} 4.744 \\ -11.82 \\ -35.505 \\ -4.744 \\ 11.82 \\ -71.038 \end{pmatrix}$$

$$v_2 := \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5.93 \times 10^{-5} \\ 1.024 \times 10^{-4} \\ -2.483 \times 10^{-3} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u_2 := v_2 = \begin{pmatrix} 5.93 \times 10^{-5} \\ 1.024 \times 10^{-4} \\ -2.483 \times 10^{-3} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Q_2 := k_2 \cdot u_2 + Q_{f2} = \begin{pmatrix} 4.744 \\ -11.82 \\ -78.962 \\ -4.744 \\ 11.82 \\ -39.235 \end{pmatrix} \qquad F_2 := Q_2 = \begin{pmatrix} 4.744 \\ -11.82 \\ -78.962 \\ -4.744 \\ 11.82 \\ -39.235 \end{pmatrix}$$



Reactions