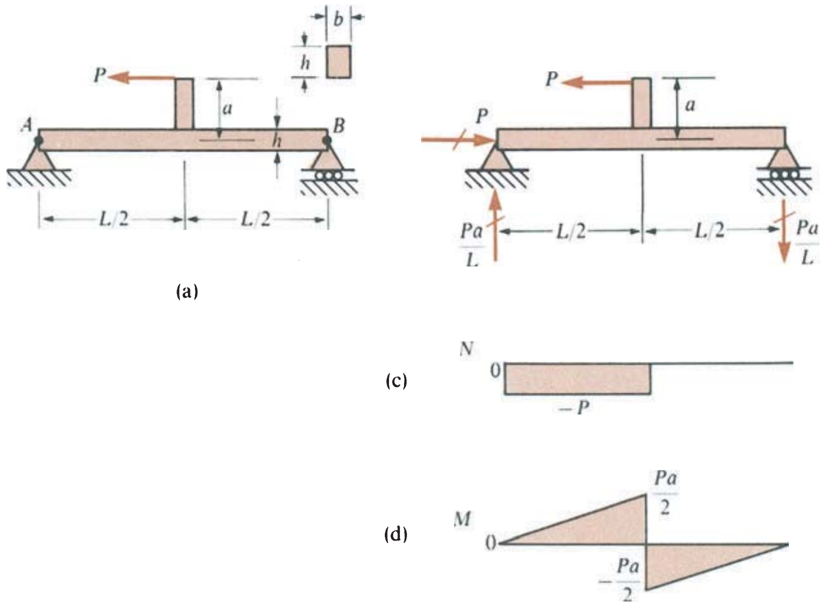


Fig. 5-39 Example

Similarly, the stresses at a cross section just to the right of the center of the beam (where $M = -Pa/2$ and $N = 0$) are

$$\sigma_b = -\frac{3Pa}{bh^2} \qquad \sigma_t = \frac{3Pa}{bh^2}$$

Upon comparing these four stresses, we see that the maximum tensile stress in the beam occurs at the top to the right of the midpoint and the maximum compressive stress occurs at the top to the left of the midpoint. These stresses are, respectively,

$$\sigma_{\text{tens}} = \frac{3Pa}{bh^2} \qquad \sigma_{\text{comp}} = -\frac{P}{bh} - \frac{3Pa}{bh^2}$$

In this example, the compressive stress is numerically larger than the tensile stress.

ECCENTRIC AXIAL LOADS

An important case of practical interest occurs when a bar is subjected to an axial load applied eccentrically, as illustrated in Fig. 5-40. In this example, the tensile load P acts normal to the end cross section at a distance e from the z axis, which is a principal axis through the centroid C (Fig. 5-40b). As in previous discussions, the y axis is an axis of symmetry.

The eccentric load P is statically equivalent to a force P applied at the centroid plus a couple Pe. Therefore, the normal stress at any point in a cross section (from Eq. 5-37) is

$$\sigma = \frac{P}{A} + \frac{Pey}{I} \tag{5-38}$$

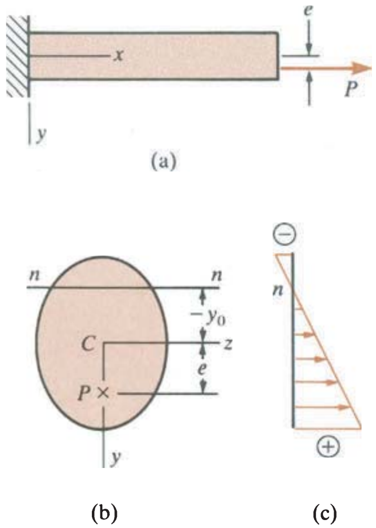


Fig. 5-40 Bar subjected to an eccentric axial force P

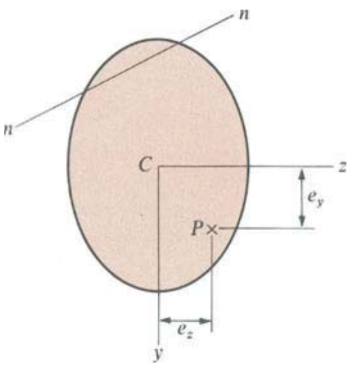


Fig. 5-41 Eccentric axial force P producing bending about both centroidal principal axes

This stress distribution is shown in Fig. 5-40c. If the axial load is compressive, the value of P in Eq. (5-38) is negative.

The line in the cross section where the stress is equal to zero (line *nn* in Fig. 5-40b) is the neutral axis of the beam. The position of this line can be obtained from Eq. (5-38) by setting the normal stress σ equal to zero and then solving for the coordinate y, which we now denote as y_0. The result is

$$y_0 = -\frac{I}{Ae} \tag{5-39}$$

The coordinate y_0 is measured from the z axis (which is the neutral axis under pure bending) to the line *nn* of zero stress (the neutral axis under combined bending and axial load). Because y and y_0 are positive downward in Fig. 5-40, the minus sign in Eq. (5-39) means that the neutral axis lies above the z axis. (Note that the load P is tensile and that the eccentricity e is assumed to be positive when P acts below the z axis.) If the eccentricity e is increased, the neutral axis will move toward the centroid; if e is reduced, the neutral axis will move away from the centroid.

When the point of application of the eccentric load P is not on one of the principal axes of the cross section, there will be simultaneous bending about both centroidal principal axes. Denoting the coordinates of the point of application of P by e_y and e_z (Fig. 5-41), we see that the bending moments about the y and z axes are numerically equal to Pe_z and Pe_y, respectively. The resultant normal stress σ at any point in the cross section (a point defined by coordinates y and z) then becomes

$$\sigma = \frac{P}{A} + \frac{Pe_z z}{I_y} + \frac{Pe_y y}{I_z} \tag{5-40}$$

where I_y and I_z are the moments of inertia about the y and z axes, respectively. In Eq. (5-40), the axial force P is positive if it is tensile, and e_y and e_z are positive in the coordinate directions shown in Fig. 5-41. Equation (5-40) reduces to Eq. (5-38) when P lies on the y axis and e_z equals zero.

The equation of the neutral axis can be found by setting the normal stress σ equal to zero in Eq. (5-40). With the corresponding coordinates denoted as y_0 and z_0, that equation becomes

$$\frac{Ae_y}{I_z} y_0 + \frac{Ae_z}{I_y} z_0 + 1 = 0 \tag{5-41}$$

This equation is linear in y_0 and z_0, and therefore the neutral axis is a straight line, such as line *nn* in Fig. 5-41. The intercepts of line *nn* with the y and z axes can be found by setting z_0 and y_0, respectively, equal to zero in Eq. (5-41) and solving for the intercepts. Then the neutral axis can be drawn as a straight line through those two points. The neutral axis may or may not intersect the cross section, depending upon the shape of the cross section and the position of the load P.

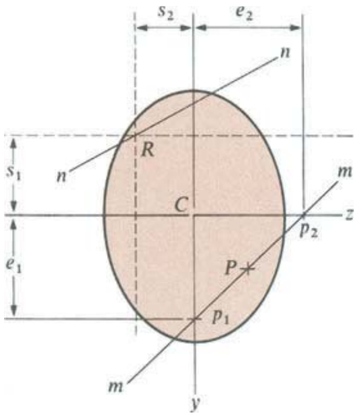


Fig. 5-42 Relationship between position of load P and neutral axis nn

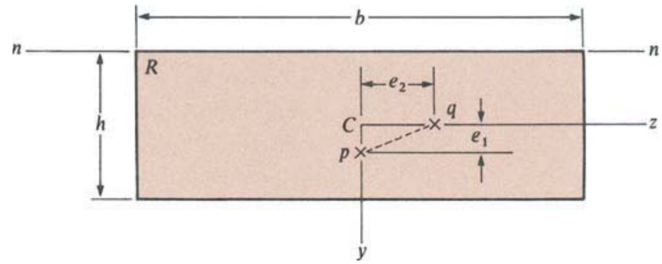
An interesting relationship exists between the point of application of the eccentric axial force P and the position of the neutral axis nn, as follows. If the force P moves along any straight line mm (Fig. 5-42), the neutral axis rotates about a fixed point R. To demonstrate this fact, we observe first that the force P can be resolved into two parallel components, one acting at p_1 and the other at p_2. The component at p_1 acts in a principal plane of bending; hence the corresponding line of zero stress is parallel to the z axis and is located at distance $s_1 = I_z/Ae_1$ from the z axis (see Fig. 5-42 and Eq. 5-39). Similarly, the component at p_2 produces bending about the y axis, and the line of zero stress is located at distance $s_2 = I_y/Ae_2$ from the y axis. Point R, at the intersection of the two dashed lines in the figure, will always be on the neutral axis nn when both components of load act simultaneously. As the load P moves along the line mm, point R remains fixed in position, and the neutral axis always passes through it. We will now use this relationship to establish the core of a rectangular section.

THE CORE OF A CROSS SECTION

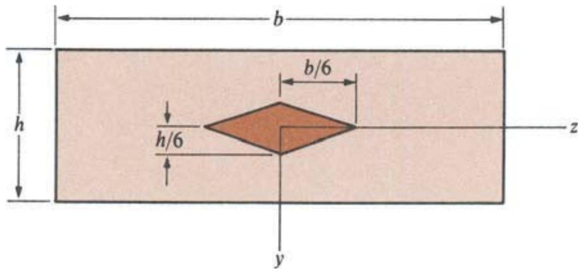
When the eccentricity e of the axial load P (Fig. 5-40) is small, the neutral axis will lie outside the cross section and the normal stresses will have the same sign throughout the cross section. A condition of this kind is important, for instance, when a compressive load acts on a material that is very weak in tension, such as glass, concrete, stone, and ceramic materials. For such materials, it may be necessary to ensure that the load produces no tension at any point of the cross section. This condition exists if the load remains within a certain small region surrounding the centroid. A compressive force acting within that region produces compression over the entire cross section, and a tensile force acting within that region produces tension over the entire cross section. This region is called the **core** (or the **kern**) of the section.*

The core of a rectangular cross section (Fig. 5-43a) can be found in the following manner. If the load lies along the positive y axis, the neutral axis nn will coincide with the upper edge of the section when the load is at point p, a distance e_1 from the centroid. The distance e_1 can be found from Eq. (5-39) by substituting $y_0 = -h/2$, $I = bh^3/12$, and $A = bh$; thus, $e_1 = h/6$. Similarly, the neutral axis coincides with the left-hand edge of the section when the load P acts on the positive z axis at point q, a distance $e_2 = b/6$ from the centroid. As the load moves along a straight line between points p and q, the neutral axis will rotate about point R at the corner of the rectangular cross section. Hence, line pq is one of the sides of the core; the other three sides can be located by symmetry. We see that the core is a rhombus with diagonals of lengths $b/3$ and $h/3$ (Fig. 5-43b).

* The concept of the core of a cross section was introduced by the French engineer J. A. C. Bresse in 1854; see Ref. 5-15.



(a)



(b)

Fig. 5-43 The core of a rectangular cross section

If the point of application of a compressive load P is within this rhombus, the neutral axis will not intersect the cross section and the entire section will be in compression. The core of other cross-sectional shapes can be found by similar techniques.

*5.10 STRESS CONCENTRATIONS IN BENDING

The flexure and shear formulas discussed in earlier sections of this chapter are valid for beams without holes, notches, or other abrupt changes in dimensions. Whenever such discontinuities exist, high localized stresses are produced. These *stress concentrations* can be extremely important when the member is subjected to dynamic loads or when the material is brittle, as discussed previously in Sections 2.9, 2.10, and 3.10.

For illustrative purposes, two cases of stress concentrations in beams are described in this section. The first case is a beam of rectangular cross section with a hole through it at the neutral axis (Fig. 5-44). The beam has height h and width b (perpendicular to the plane of the figure) and is in pure bending under the action of bending moments M. When the diameter d of the hole is small compared to the height h, the stress distribution on the cross section through the hole is approximately as shown by the diagram in Fig. 5-44a. At point B on the edge of the hole, the actual stress is much larger than the stress that would exist at that point if